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The Effects Of Teaching Exponential Functions Using Authentic Problem Solving On Students' Achievement And Attitude

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THE EFFECTS OF TEACHING EXPONENTIAL FUNCTIONS USING AUTHENTIC PROBLEM SOLVING ON STUDENTS' ACHIEVEMENT AND ATTITUDE

by

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DEDICATION

To my mother, without whom I could have never tread this path. You were and continue to be the reason I strive to be all that I can be.

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CHAPTER 1 INTRODUCTION

If we gave a student a tool box filled with great tools and taught that student the mechanism of each and every tool until the student became proficient in operating each and every one of them. That is a great accomplishment but unfortunately this knowledge is worth nothing because that student has never had the chance to use these tools to fix anything. That student was never put in a situation in which he had to figure out what tool to use and when. This kind of approach is what we commonly see in mathematics education these days. Students are taught how to manipulate variables without having any understanding of why they do it. The focus, in most of the mathematics curriculums, is on the procedural knowledge not the conceptual knowledge. Schoenfeld (1994) stated, “The current mathematics education is based on a false mastery model, in which isolated skills are taught in the hope can then be used to solve prepackaged problems” (p. 54).

Overview of the issue

Mathematical concepts are great tools when used the right way. To help ensure that students know how to use the mathematical concepts to solve problems the Common Core State Standards (CCSS) and the National Council of Teachers of Mathematics (NCTM) stressed on conceptual understanding and problem solving. In their effort to emphasize on conceptual understanding and problem solving the 2010 CCSS Mathematical standards, indicated the qualities of the mathematically proficient students as:

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- Start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.
- Explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends (p. 7).

A mathematically proficient student needs to learn more than the procedures in mathematics; he/she need to deeply understand the concept. In other words the Common Core is calling to prepare students to reason mathematically and use their knowledge as a tool to solve problems. This aligns very well with the NCTM (2000) Principles and Standards, which indicated, "Problem solving means engaging in a task for which the solution method is not known in advanced. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understanding" (p. 52). The NCTM's (2000) problem solving standards for grades 9-12, stressed on the importance of problem solving in building new knowledge "to meet the challenges in work, school, and life, students will have to adapt and extend whatever mathematics they know. Doing so effectively lies in the heart of problem solving" (p. 332). The NCTM (2000) encourages teachers to use problem solving as a mean for learning any mathematical concept "most mathematic concepts or generalizations can be effectively introduced using a problem situation that helps students see important aspects of the idea to be generalized" (p 335).

In the vast majority of mathematics books, acquiring knowledge is separated from applying it. Their focus is on teaching students the procedure. When the students master it they move to the applications. The assignment consists of applying the procedure for the majority of the problems, and then at the end, there are a number of applied problems. Such problems are only a different rendition of the previous problems because students are only required to apply the procedure. In such problems students do not have to think because everything fits perfectly into the formula and all the students have to do is follow the procedure. Jonassen (2000) indicated:

> Unfortunately, students are rarely, if ever, required to solve meaningful problems as part of their curricula. The few problems that students do encounter are normally well-structured (story) problems, which are inconsistent with the nature of the problems they will need to learn to solve in their everyday lives (p. 63).

The problems students will encounter in their real lives will have no formula given from which they may follow. Decisions made in such situations would have to be based on a true understanding of mathematical concepts and the ability to use them efficiently.

Statement of the problem

Karaçalli and Korur (2014) defined attitude as "someone's tendency to consider an object, a case, and a person in a positive or negative manner" (p. 225). Pyzdrowski et al. (2013) stated, "Attitudes can be thought of as beliefs with an added value-laden or evaluative dimension" (p. 532).

The increase in students' negative attitude toward mathematics has been noticed by a lot of researchers and educators. Curtis (2006) indicated:

> Student attitudes toward mathematics have been the focus of literature and research for decades. When students are in younger grades, they find mathematics enjoyable. However, as students progress through grade levels, their interest in mathematics begins to decline. By the time they reach college, few students pursue a mathematics degree. Others take mathematics courses only because it is a requirement to graduate from college (p. 147).

In addition to the increase of negative attitude toward mathematics, a decline in students' achievement was also noticeable, Tapia (1996) stated, "Declining national test scores in mathematics and dislike of mathematics have increased attention to students' attitudes since these attitudes are important in the students' achievement and performance" (p. 8). Pyzdrowski et al. (2013) indicated, "Negative attitudes toward mathematics are quite common among Americans, with 93% indicating that they experience some kind of negative attitude toward learning mathematics"(p. 532).

Students' attitude has been blamed for the declining national and international test scores. The negative attitude that students' have for mathematics is cited as the reason for the low achievement of such students. Edwards and Rule (2013) stated, "Attitudes are particularly important because they carry a mental state of readiness, directing the learner's attention through the educational experience and thereby affecting learning outcomes" (p. 50). Students' achievement is correlated to students' attitude. Curtis (2006) indicated, "The students' attitude toward the positive or negative learning environment had a direct correlation with learning outcomes" (p.57). In 2014, Karaçalli and Korur researched the effect of project-based learning on students' achievement and attitude. They concluded, "Students' achievement increases by increasing their desire to learn science" (p. 225). Tapia (1996) stated, "Research has indicated that attitudes toward mathematics are very important in the achievement and participation of students in mathematics" (p. 5).

In an attempt to find the reason for such negative attitude, the curriculum and the teaching method were the main two things to focus on. Using a traditional curriculum is not satisfying students' need in a world full of technology. Curtis (2006) indicated that traditional teaching methods and curriculum were not fulfilling the needs of the students. Curtis also called for changing the way teachers teach mathematics from modeling procedures and expecting students to copy their method, to helping students construct their understanding of the concept using problem solving. Chang (2010) mentioned, "Repeatedly showing students what we want them to know will not automatically help students to translate the knowledge into their own" (p. 247). Alsup (2005) indicated, "The teacher cannot transmit mathematical knowledge directly to students, but students construct it by resolving situations they find problematic" (p. 1).

The call for change is been focused on using problem solving to teach different mathematical concepts. Fennema et al. (1996) stated, "The gains in students' concepts and problem-solving performance appeared to be directly related to changes in teachers' instruction" (p. 430). Problem solving does not only make mathematics concepts relevant to students, it also helps students construct their understanding in a way that cannot be achieved using the traditional approach. Problem solving is very beneficial to students; NCTM (2000) indicated, "By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom" (p. 52). Problem solving helps them feel that they can use their knowledge and not just follow a memorized procedure. The feeling that they can use their knowledge to achieve something will improve their achievement.

Wang and Posey (2011) concluded, "Using problem solving helped students to become more independent, increased their confidence in their problem solving ability, and gained a deeper conceptual understanding" (p. 494).

Significance of the study

Authentic problem solving. For a mathematics problem to be remembered by students, it must be meaningful and relevant to their lives. Hiebert et al. (1996) indicated that the value of a mathematics problem is by the residue it leaves "of course, important mathematical residue can be left by grappling with real-life problems" (p. 18). The reason real life problems leave an important residue is because they connect to students' lives. It is not possible for a student to remember a concept if it was not meaningful and of value. For the concept to be of value it must be used to solve a real life problem. Jonassen (2000) point out that "most psychologists and educators regard problem solving as the most important learning outcome for life. Why? Virtually everyone, in their everyday and professional lives, regularly solves problems" (p. 63). It is clear that for the mathematical concepts to be understood and valued, it must be presented using a meaningful method. Using authentic problem solving is what gives life to the problem and moves it further away from abstraction. Hiebert et al. (1996) stated, "Mathematics acquired in these realistic situations will be perceived by students as being useful. Rather than acquiring knowledge that is isolated from real situations, students will acquire knowledge that is connected to such situations, and they will be able to apply this knowledge to a range of real-life problems" (p. 14).

Problem solving and students' achievement. Teaching for understanding is the ultimate goal of both mathematics educators and mathematics education organizations. Every concept that students learn with understanding will equip them with an additional tool to help solve uprising problems in their lives. Learning with understanding will help them not only know how to use the concept, but also they will understand when and why they will use it. This will expand their horizon and add depth to their understanding of the mathematics they encounter, which will positively improve their mathematical achievement. Lindquist (1997) indicated, "Understanding is crucial because things learned with understanding can be used flexibly, adapted to new situations, and used to learn new things. Things learned with understanding are the most useful things to know in a changing and unpredictable world" (p. 1). Understanding can be achieved when students are engaged in their learning and through solving problems. To achieve the ultimate goal of teaching with understanding, problem solving is the most appropriate method to attaining that goal. According to Hyde and Hyde (1989) "Problem solving is a way of thinking and doing mathematics, it can be a major vehicle for helping students truly understand and use mathematics. Problem-solving activities can introduce students' new mathematical ideas; provide exciting experiences that develop deeper understanding" (p. 5). By using problem solving to teach mathematical concept, educators will deepen students' conceptual understanding. This kind of understanding is the key to help improve students' achievement and performance.

Problem solving and students' attitudes. Students' attitudes play a crucial factor in their learning. If they view a topic as important and relevant to their lives, they will have a positive attitude and will work hard to learn it. Teaching mathematics using problem solving helps improve students' attitudes and understanding of the concept. As stated by Seligman (2007), "For the students in this study, both their attitudes and understanding of mathematics improved as a result of solving non-routine problems." (p. 113). Curtis (2006) also indicated, "All four of the teaching pedagogies, cooperative learning, graphing calculators, discourse, and problem solving had an impact on the attitudes of students in mathematics. Each strategy played a role in making the learning environment in this classroom a better experience for the students" (p. 156).

When students relate what they are learning to their lives, they become more active in the learning process. Using problem solving to teach helps students understand the concept in addition to understanding when and how math is used in real life. This understanding will be reflected in their attitudes. Wade (1994) indicated, "Qualitative data revealed that the constructivist based mathematics problem solving instructional program caused a positive shift in student attitudes toward mathematics problem solving" (p 119).

Exponential functions. Exponential function is an abstract topic in Algebra. Most of the Algebra books, that I encountered, dedicated a couple of sections to simplifying exponents, one section for graphing exponents and one for exponential functions. The exponential function is taught by giving the students the general form of the exponential equation (y$= Pr^t$, P is the principal, r is the rate of growth or decay, and t is the time), then students practice writing exponential functions using the formula. The students will finish this section in about two days. They will only know how to plug in the numbers instead of the designated variable to get the final answer. They will barely have any knowledge of why. Exponential functions will never be seen in Algebra 1 again. The students will encounter a similar approach and be exposed to exponential functions in Algebra 2. Students don't value this topic, the way it is taught currently, and most of the times they forget everything related to it after the test. The topic was abstract and irrelevant to their lives. Connected Math (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998) was the only curriculum I encountered that dedicated a chapter to construct students' conceptual knowledge of exponential functions. The CCSS wants students to understand the concept of exponential functions and use it to represent real life situations. They expect students to "recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another" (CCSS, p. 70). They also want students to represent a given situation as a function "construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs" and "interpret the parameters in a linear or exponential function in terms of a context"(CCSS, p. 71).

Exponential functions can be observed a lot in our world; the division of the cell, population growth, half-life, medication in the body, and compound interest. When students are given the chance to explore, discuss, and write exponential equations to represent these situations, exponential functions become meaningful and important. This approach will help students understand it, value it, and relate it to their world. This couldn't be achieved if they memorized and applied the rule. To help students understand the concept of exponential functions, it should be introduced to them through problem solving and real-life situations.

It is very clear that our current approach in teaching mathematics is not producing the mathematical understanding that we desire. The major mathematics organizations as well as a lot of educators are calling for a change in the approach mathematics is being taught. They want to balance the focus between procedural knowledge and conceptual knowledge. In order for conceptual knowledge to be of value it must be taught through problem solving. Problem solving is a great teaching method to introduce and deepen students' understanding in all mathematical concepts. For the acquired knowledge to be permanent students' must value it. To help them value it, we need to let them experience it by using it to solving problems. For the great mathematic tools to be of value and reach its desired outcome students must be exposed to situations in which they can understand how to use the tool as well as when and why the specific mathematics tool was chosen.

This research will explore and measure the effect of problem based curriculum on students' achievement and attitude.

The research questions are:

1. How is students' achievement affected by the use of authentic, problem-based learning when teaching exponential functions?
2. How is students' attitude affected by the use of authentic, problem-based learning when teaching exponential functions?

CHAPTER 2 REVIEW OF LITERATURE

This chapter will provide a review of the literature focusing on the problem of the way mathematics is currently taught and on the proposed solution of using problem solving to teach mathematics. The problem with the mathematics education in the U.S. lies in the way mathematics is taught. The focus, most of the time, is on procedural understanding in which all the student have to do is memorize a set of rules for the purpose of repeating them in the test. The student will have no future use of such knowledge; that's why it will eventually be forgotten shortly. The students are not challenged and are not asked to use their knowledge in a situation beyond what they learned in the unit.

Educators and major organizations such as NCTM are calling for a change. The type of change that focuses on helping students makes sense and deeply understands the concept. This change will require shifting students learning and understanding from only procedural knowledge to include conceptual understanding. The dilemma is conceptual understanding cannot be taught without active engagement, students must be actively involved in making sense and understanding the concept. The best approach to achieve this kind of understanding is by using problem solving as a tool to teach all mathematics concepts. Since not all problem solving have the same effect on students, authentic problem solving is the solution for two reasons; it is relevant to their lives and it helps them experience the benefits of what they learn in mathematics in solving real life problems. The mathematics topic that I specifically focused on is exponential functions. Exponential functions are visible in the growth of bacteria, interest rate, the decay of medicine in the blood stream, and half-life of carbon. To help students fully understand these functions, they need to actively explore them through solving real life problems.

This chapter is divided into a few sections the first section being about the challenges facing the current mathematics education system in the United States. The second section will focus on making sense and understanding, and the third section will be about using problem solving to teach mathematics. The fourth section will provide literature about teaching exponential functions, and the last section will focus on the effects of using problem solving on students' attitude.

The challenges with current mathematics education system in the United States

To understand and to evaluate students' mathematics performance in the U.S. we need to look at the national data and compare it to other countries. The Trends in International Mathematics and Science Study (TIMSS) and the Program for International Students Assessment (PISA) provide a good feedback to mathematics educators in terms of comparing the mathematics performance of the U.S. students to the rest of the world.

According to the National Center for Education Statistics, the U.S. averaged in the TIMSS was 492 in 1995, 508 in 2005, 509 in 2011, and 518 in 2015. This is slightly above the international average but lower than a number of developed countries. The U.S. students are still lacking behind in their mathematical performance. Hanushek, Peterson, and Woessmann (2010) indicated that:

> No less than 30 of the 56 other countries that participated in the Program for International Student Assessment (PISA) mathematics test had a larger percentage of students who scored at the international equivalent of the advanced level on our National Assessment of Educational Progress (NAEP) tests (p. 4).

This is a clear message that our system does not focus on the depth of the mathematical knowledge rather the focus on the procedural knowledge. The most important thing is to finish the required material but not to emphasize the depth of students' understanding.

The mathematical education system we currently have does not produce highly mathematics proficient students. Hanushek, Peterson, and Woessmann (2010) indicated that Taiwan, Hong Kong, Korea, and 12 other countries had at least double the percentage of highly proficient mathematics students as the U.S., "In short, the percentages of high-achieving mathematics students in the U.S.—and most of its individual states—are shockingly below those of many of the world's leading industrialized nations" (p. 4). From my own experience as teacher, there is no doubt in my mind that we have a good percentage of very smart students but our mathematical system is failing, and it is not helping them reach their potential. The current system focuses on teaching a procedure, practice it with them a couple of times, give them a number of problems to practice on their own, and finally give them a well behaving word problem with everything clear for them to follow the same procedure. Bellamy and Mativo (2010) suggested "Not all students necessarily learn in a classroom focused on standardized tests, most 'sit, spit, and forget" (p. 27).

The challenges facing the mathematics education system is not only clear to educators; it seems that it is very dramatic that everybody knows about it. According to Hanushek, Peterson, and Woessmann (2010), "The public seems to have grasped the fact that American students are faltering in math and science relative to their peers in other countries." (p. 7). They also indicated that businesses are complaining from the shortage of skilled works, "The gap between the burgeoning business demand for a highly accomplished workforce and a lagging educational system has steadily widened" (p. 7). Schoenfeld (1994) agreed with Hoffman (1989) when

Hoffman pointed out that the current mathematics education system focuses on teaching isolated skills with the hope that these skills will bond together to help solve problem. The problem lies in an education system that focuses on teaching students isolated skills without focusing on the big picture hoping that the students will connect the skills together to form a better understanding of the use and properties of each of the skills they learned. The focus is on the procedural knowledge not on the conceptual understanding.

With all of these challenges in the mathematical educational system all the eyes are focused on the National Council of Mathematics Teachers (NCTM) and Common Core State Standards (CCSS) to propose a solution. Ferrini-Mundy and Schmidt (2005) "the mathematics education community needs to be strongly represented in all aspects of these initiatives if such studies are to realize their potential are resources for improving mathematics teaching and learning" (p. 173).

In an effort to improve mathematics education and student achievement, the NCTM vision (2000) stated that the need "Where all students have access to high-quality, engaging mathematics instruction" (p. 3). As we continue falling behind in our students' mathematics abilities we are realizing more that there is a problem in the way mathematics is taught in the U.S. It is clear that the way mathematics education needs a complete makeover to produce students with abilities that can be a head in such international tests, help them be ready for future challenges, and prepare them to solve problems that will face them at work and in their lives. Cavanagh (2005) states, "to some education experts, though, the U.S. performance on the two international exams reinforced their belief that American students suffer from an inability to perform complex reasoning and mathematical assignments - the kind they are likely to encounter in college and the workplace" (p. 1).

The CCSS (2010) echoed this need for better education preparing for college and workplace and indicated that there is a problem in the mathematics education, “Research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country” (p. 3).

Making sense and understanding

Reasoning and making sense. The goal for any mathematics educator is to help students use mathematics effectively in their lives. This goal cannot be realized if the students learn how to follow a given recipe without understanding and making sense of the concept. This goal can be achieved when students understand and make sense of the mathematics as they learn and experience doing mathematics in school. When students understand and make sense then when faced with a mathematics problem, they will know what mathematics tool to use and how to use it. To help students learn with understanding new methods of teaching and assessing must be implemented.

In 2009, Martin et al. defined reasoning as “the process of drawing conclusions on the basis of evidence or stated assumption” (p. 4). The NCTM (2000) also defined sense making as "developing understanding of a situation, context, or concept by connecting it with existing knowledge (p. 4). Martin et al. (2009) called reasoning and sense making the cornerstone of mathematics and they suggested the restructuring of the mathematics program around these two concepts, they stated, “Restructuring the high school mathematics program around them enhances students’ development of both content and process knowledge they need to be successful in their continuing study of mathematics and in their lives” (p. 7).

To help students understand and make sense teachers must use new teaching methods and move away from the traditional methods of lecturing and giving tests. In a research conducted by Pinzker (2001) to evaluate the effect of replacing the traditional method of teaching by using cooperative learning, alternative assessments, and journal writing on students achievement and understanding, she found that “the number of achieving students earning an A, B, or C increased from 19 students to 26 students, a 36% increase” (p. 27). In the same study a portfolio was used to measure students’ understanding instead of teacher-created tests. The use of portfolio had a positive effect on students’ understanding; Pinzker indicated, “The use of portfolio also had a positive effect on students understanding of mathematical concepts. The number of achieving students earning an A, B, or C on their portfolio was 27 out of a total of 30” (p. 28). To help improve students’ achievement, the focus must be on students’ understanding and sense making which will require educators to navigate away from the traditional methods of lecturing and testing.

Collaboration and discussion between students while solving a problem helps them make sense and understand the mathematics better. One of the goals of a study conducted by Adamson in 2005, with students enrolled in an intermediate algebra class, was to understand the effect of students’ collaboration and discussion on their sense making of the concept of the slope. Adamson used five different methods to evaluate and measure students’ sense making and understanding. It was clear the classroom environment and student interaction helped students make sense of what they are learning. Adamson concluded, “Student sense-making about slope requires multiple opportunities and much time for student understanding to result. I found that a classroom environment that supports classroom discourse can aid students in their sense-making efforts” (p. 207).

The idea of teaching students in a way that helps them reason and make sense of the mathematical information was the focus of mathematics educators and major mathematics organizations. They did their best to lay the foundation for the mathematics teachers in order to educate students in a way that would help them reason and make sense of what they are learning. The NCTM's Principles and Standards (2000) indicated, "Being able to reason is essential to understanding mathematics" (p. 56). The NCTM stressed on the importance of learning with understanding because it enables students to use their knowledge to solve problems they will face in their work and in their lives. The NCTM stated "students will be served well by school mathematics programs that enhance their natural desire to understand what they are asked to learn" (p. 21). The CCSS (2010) vision of mathematically proficient students is for students to make sense of problems, reason abstractly and quantitatively, and critique the reasoning of others. The standards by NCTM and CCSS, two reform documents, emphasize the focus to shift from procedural knowledge to teaching with understanding and making sense of what is learned so that the acquired knowledge can be applied and expanded to solve problems they might encounter at work or in their lives.

Most of the mathematics taught today is totally the opposite of the NCTM and CCSS vision. In almost all grade levels the focus is on the procedure and not on understanding. Pesek and Kirshner (2000) indicated that students who learned to focus on procedures will focus on manipulating variables. The main goal for these students becomes knowing how to solve a problem not why. In other words they do not focus on conceptual understanding but all their focus is on the procedure. The problem with this kind of understanding is that they did not reason or make sense of what they were doing, which makes their knowledge shallow and easy to forget. Martin and Kasmer (2010) state:

> Reasoning and sense making in mathematics are fundamental to children's ability to learn and know mathematics. The central goals of all grade levels should shift from an emphasis on memorizing procedures and algorithms to an emphasis on exploring important mathematical ideas, making conjectures, drawing conclusions, and forming generalizations. Such an emphasis ensures that students will have a robust knowledge of mathematics that they can apply in a variety of contexts. It also provides motivation by stimulating their natural curiosity (p. 286)

Fernandez conducted a study in 1994 to evaluate the effect of two types of instructions on the way students learn. The two methods of teaching indicated by Fernandez are “one which I called sense-building and which emphasizes student thinking and sense making, and the other which I called skill-teaching and which emphasizes student acquisition of procedures being taught by the teacher” (p. 66). Ninety nine fifth and sixth graders were randomly divided into two groups; one group watched a lesson that emphasizes on sense making and understanding while the other group watched a lesson that emphasizes on procedural knowledge. Fernandez expected that the sense-building lesson will focus students’ attention on understanding and making sense, while skills-building will focus students’ attention on procedures. Fernandez concluded that:

> the children who watched the skills-building lesson were better able to carry out the procedures being taught, their acquisition of these procedures does not seem to have been grounded in having processed the lesson by trying to understand it. The children who watched the sense-building lesson on the other hand seem to have been more actively trying to understand the instructions by attempting to form a coherent representation of it (p. 68).

To help students make sense and understand, the teaching method must reflect the sense making and understanding. Most of the students who learn to carry out procedures without adequate focus on the concept will eventually forget the procedures. If the purpose of learning mathematics is to equip students’ with the tools to help them solve problems, then focusing only on the procedure is not the best way to teach mathematics. Students must understand and make sense of the concept so they are able to solve the problems they will face in their future.

Teaching mathematical concepts through problem solving is the recommended way to prepare students for the upcoming challenges. In a study conducted by Miller and Fey (2000) to determine the effect of using problem-base learning on students' proportional reasoning skills, they indicated, "Students who had worked through a program emphasizing discovery of concepts and skills through problem-based teaching developed much stronger proportional-reasoning skills than students who were taught using conventional curricula and direct teaching approaches" (p. 312).

Hiebert et al. (1997) believed, "In order to take advantage of new opportunities and to meet the challenges of tomorrow, today's students need flexible approaches for defining and solving problems" (p. 1). This approach equips students with a flexible knowledge that can be used to solve any given situation. This kind of knowledge cannot be acquired by focusing on the procedures. Knowing how to manipulate and move variables around is not the required knowledge for the twenty first century. The required knowledge is the knowledge that helps students understand and make sense of what they are learning. Hiebert et al. (1997) indicated, "Understanding is crucial because things learned with understanding can be used flexibly, adapted to new situations, and used to learn new things" (p. 1). In an attempt to understand how students make sense of word problems Nosegbe conducted a study in 2001 to examine the effect of a two week instructional program on students understanding and making sense of word problems. The instructional program was designed to help students improve their understanding and making sense of real life problems. A pre-and a post-test were administrated to 76 sixth-grade students. 18 students were randomly chosen for interviews. Both of the tests consist of carefully chosen problems to measure understanding and sense making. After analyzing the pretest and posttest Nosegbe concluded, "Students' performance improved in all categories (strategy used, answer given, and justification provided) as well as in their overall performance" (p. 83). She stated, "The improvement in students' scores validates my assumption that students' problem-solving performance would increase after instruction" (p. 81). Students' understanding and sense making of the problem also improved which was observed during the interviews. Nosegbe indicated, "After the instruction had been implemented, the percentage of students who accurately justified their problem-solving answers during the posttest was significantly higher than the percentage during the pretest" (p. 130).

Martin et al. (2009) stated, "a focus on reasoning and sense making when developed in the context of important content, will ensure that students can accurately carry out mathematical procedures, understand why these procedures work, and know how they might be used and their results interpreted" (p. 3). This kind of understanding will equip the students with the ability to expand their mathematical knowledge and apply it to different situations. Carpenter and Lehrer (1999) described this kind of learning as generative: "When students acquire knowledge with understanding, they can apply that knowledge to learn new topics and solve new and unfamiliar problems" (p. 21). Carpenter and Lehrer (1999) warned from the effect of learning mathematics without understanding, they indicated, "When students do not understand, they perceive each topic as an isolated skill. They cannot apply their skills to solve problems not explicitly covered by instruction, and neither can they extend their learning to new topics" (p. 21). It is not possible to teach the students all the skills and give them examples of all the problems they will encounter. Without understanding making sense of what students learn, their knowledge will not be useful and it will be forgotten shortly after they finish their mathematics requirements.

Procedural and conceptual knowledge. Understanding and making sense of the mathematics is essential for students' success in using what they learned to solve problems. This understanding does not mean only knowing how to follow the procedure but it also includes understanding the concept, which include knowing why a certain procedure is being followed and when to use that procedure. When a student understands and makes sense of a mathematical concept then it becomes natural to apply a procedure at the right time and to interpret results correctly. According to Martin et al. (2009), “Sense making and conceptual understanding are closely interrelated” (p. 12). Making sense cannot be achieved if students do not understand the concept, and conceptual knowledge will not be acquired without making sense of what they learn. To achieve making sense and understanding students need to understand the concept not only apply the procedure. If a student applies the procedures correctly that does not indicate that conceptual understanding was also achieved. In most of the current mathematics classrooms, students memorize and carry out a number of procedures without fully understanding the concept. When such students are faced with a problem in which they have to choose the correct procedure they get confused and a lot of them fail to choose the proper procedure. In a study conducted in 1999 by Dickerson to compare traditional approach and problem posing approach. Dickerson stated:

> Unlike traditional mathematics instruction that reduces most student understanding to rote application of memorized formulas, problem posing provides students with the opportunity to think and reason logically, to design and test conjectures, to make rational sense of mathematics and the real world, and to justify personal understanding (p. 75).

For students to learn with understanding, teachers must focus on helping them make sense of the mathematics they learn by making sure that they understand why they are using each procedure and when to use it. Dickerson (1999) suggested that “students learn best when they build their own knowledge out of active sense-making activities that relate their current understanding to new situations" (p. 71).

Procedural knowledge and conceptual knowledge are two different types of knowledge, yet both types of knowledge are necessary for students. Rittle-Johnson and Alibali (1999) defined conceptual and procedural knowledge:

> We define conceptual knowledge as explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain. We define procedural knowledge as action sequences for solving problems. These two types of knowledge lie on a continuum and cannot always be separated; however, the two ends of the continuum represent two different types of knowledge. (p. 75)

Both types of knowledge need to be the focus when teaching mathematics. Both types are needed to help students understand and make sense. When teachers focus on one type during the lesson the other type of knowledge will not be fully achieved. Deslauriers conducted a study in 2008 to compare the effect of teaching in two different ways: conceptually-focused instruction (CFI) and procedurally-focused instruction (PFI). Twenty-four high achieving ninth graders were divided into two identical groups in which each two students were paired according to their pretest score, one of them was randomly placed in the CFI and the other in the PTI group. Deslauriers indicated, "The findings showed that both types of instruction were beneficial for students. Students in CFI group developed conceptual understanding with brittle skill acquisition and students in PFI group developed problem-solving skills with fragile understanding of concepts" (p. 199). For students to succeed they need to understand the concept of what they are learning and be fluent in the procedures. Deslauriers advised the use of the two approaches in teaching, she stated that "the results imply that transfer of cognitive knowledge representations and processes occurred differently in the two conditions of focused instruction, more readily and sustained in CFI, and more gradually in PFI" (p. 199).

For students to make sense and understand the mathematics they learn in school they need conceptual knowledge and procedural proficiency. Martin et al. (2009) stated that "Procedural fluency includes learning with understanding and knowing which procedure to choose, when to use it, and for what purpose" (p. 12). When students know what procedure to use, when and why they use it, they actually achieve understanding and they make sense of what they have learned.

Conceptual knowledge and procedural knowledge are interconnected; students need to be fluent in both to be able to expand their knowledge to solve problems. In a study conducted by Rittle-Johnson and Alibali in 1999 to understand the relation between conceptual and procedural knowledge, they found that there is a strong relation between procedural and conceptual knowledge; improving one type of knowledge will help improve the other. Rittle-Johnson and Alibali (1999) claimed:

> Conceptual instruction led to generation of correct, flexible procedures for solving equivalence problems, and procedural instruction led to gains in conceptual understanding. Thus, the relations between conceptual and procedural knowledge are not unidirectional. Instead, conceptual and procedural knowledge appear to develop iteratively, with gains in one type of knowledge leading to gains in the other (p. 188).

In the same study, Rittle-Johnson and Alibali in 1999 found out that the two types of knowledge don't affect each other evenly; the influence of conceptual knowledge on procedural knowledge is more than the influence of procedural knowledge on conceptual knowledge. Rittle-Johnson and Alibali (1999) indicated that:

> Gains in conceptual understanding led to fairly consistent improvements in procedural knowledge in this study. Children who received conceptual instruction were just as likely to learn a correct procedure as children who received procedural instruction, and conceptual instruction led to better transfer performance than procedural instruction. Thus, conceptual knowledge seemed to have a greater impact on procedural knowledge than the reverse. (p. 186).

Rittle-Johnson and Alibali (1999) suggested that "teaching children the concept behind mathematical equivalence problems, rather than a procedure for solving them, was most effective at promoting flexible problem-solving skill and conceptual understanding" (p. 188).

In most mathematics classrooms the emphasis is on procedural knowledge with barely any attention to conceptual understanding. The result is students who know the procedure and they apply it multiple times but have very little understanding of why are they using the procedure. In 2006, Clark conducted a study to examine whether using a conceptual approach to teach functions will improve 134 students' understanding and achievement better than using a procedural approach of teaching functions. The two groups were chosen such that there was no significant difference between their achievements in the pre-test. Clark indicated, "The posttest revealed that conceptual students had a better overall understanding of the function concept than procedural students" (p. 44).

When procedures are taught and practiced in separation of the concept, it become very difficult for students to arrive to the concept on their own. The concept must be clear from the beginning. Hiebert et al. (1997) believe that the reason for this separation is the traditional instructions system and that "by trying to teach concepts and procedures directly, we artificially separate them. Although we may try to get students to hook them back together, this is more difficult than we think and most students are not successful" (p. 25). When the focus is only on procedural knowledge, students learn by following the steps provided to them by their teacher. Their focus becomes knowing how to carry on the steps of a given procedure not why. After they practice the procedure they are asked to understand the concept by solving a couple of problems that targets the conceptual knowledge. It could be very difficult to glue the two together. After they know how to follow the steps and the knowledge is constructed, it might be very hard to attach the conceptual understanding to it.

Both types of knowledge need to be constructed together. Teachers are encouraged to navigate away from the traditional methods of teaching to use new methods that allow the construction of the two type of knowledge. In 2006, Ross conducted a study to evaluate the effect of student-centered approach on students' conceptual and procedural knowledge and therefore their achievement. She concluded, "The results from the final structural equation model revealed a significant correlation at the within level for the variables of student procedural knowledge and student conceptual understanding" (p. 96). Ross advised teachers to plan lessons that help students construct both types of knowledge simultaneously. In the experimental group students were given the opportunity to be involved in constructing their understanding. Ross indicated, "This engagement revealed increased understanding, as evidenced through increased higher-level discourse" (p. 100).

Conceptual understanding and procedural fluency are the main components to success. Both are crucial in helping students make sense and understand what they learn. Store, Berenson, and Carter (2010) note that "algebraic reasoning goes beyond simply manipulating algebraic symbols" and that this in turn challenges teachers to "transition from the traditional teacher-centered practices to teaching practices that make algebraic reasoning more accessible to students" (p. 52). One of the suggested ways to help students construct their understanding of both types of knowledge is using problem solving to teach. Hiebert et al. (1997) suggested that teachers should pose a problem and encourage students to develop their own procedure to solve it. A study was conducted in 2008 by Boaler to determine the effect of using problem solving and group work on students' achievement. She followed approximately 700 students for four years. The students were in three different high schools, two of which used the traditional teaching method while the third school used problem solving and group work. The teacher in the third school posed a problem that requires conceptual understanding and gives the students the time to solve it. The teacher guides their work and helps any group that asks for help. At the end of class students present their solution. Boaler indicated that at the beginning of the study, the students in the third school were achieving significantly lower than the other two schools, however "within two years they were out-performing the other students, scoring at significantly higher levels on mathematics tests" (p. 10). Boaler indicated that by the fourth year, 41% of the seniors were in advanced pre-Calculus and Calculus classes while only 27% of the seniors in the other two traditional schools were enrolled in such classes. This method of teaching forces the students to use what they know to construct new procedure. In this way the procedure is based on the conceptual understanding. Students will construct their conceptual understanding as well as their procedural fluency in a way that makes sense to them. This kind of construction enables them to expand their knowledge.

Using problem-based learning with the focus on conceptual understanding help students activate their prior knowledge and construct their new understanding in a way that can be used to solve similar problems. As Deslauriers (2008) indicated, "Problem-based learning of high quality (theory-driven) with a conceptual focus may be more effective for novice and intermediate students in mathematics" (p. 200).

Using problem solving to teach mathematics

Problem solving. Using problem solving to teach mathematical concepts enables students to construct their conceptual understanding as well as their procedural fluency. It will help students understand when and how to use a certain concept to solve problems that they might encounter in the future. This understanding will be reflected in their achievement and their ability to solve problems in their lives and in the future. Using Lester and Mau's (1993) definition of teaching via problem solving, Bull (1993) defined it as:

> a change from viewing teaching as an act of transmitting information to passive students to an act of helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying. This change in point of view requires a correspondingly fundamental change in the teacher's role in the classroom. Rather than being the ultimate authority and dispenser of knowledge, the teacher variously plays the role of guide, coach, question asker, and co-problem solver (p. 18).

Solving problems helps students deepen their understanding of the mathematical concept. It is different from the traditional method of teaching. This kind of understanding enables them to connect the concept with its use and benefits. King (2005) indicated, "While the teaching of skills and strategies is important, in order to develop mathematical thinkers, teachers should consider using problem solving as a part of the classroom practice to assist in the development of mathematical thinkers" (p. ii). King advises teachers to deviate away from the traditional method of giving the information to students and replace it with methods that involve students in the exploration of new concepts.

Using problem solving positively affects students' achievement. In a study conducted by Bull in 1993, to evaluate the effect of using a four-step problem solving method on students' achievement, students were randomly divided into two groups; the experimental group consisted of 274 and the control group consisted of 237 students. The control group used the traditional teaching method while the experimental group used problem solving techniques. The students were tested before and after the experiment using the SAT-8 test. Bull indicated, "Students that were taught with an emphasis on the four-step method of problem-solving, a method that addresses the personal strengths of students, improved significantly more in mathematics than students taught through more traditional methods" (p. iv). Bull advised that "all mathematics should be studied in contexts that give the ideas and concepts meaning. Problem solving should arise from situations that are not always well formed" (p. 46). He recommends using problem solving because "For students to function effectively as adults in today's society, they must know and be able to use mathematics in both their personal lives and their professional lives" (p. 49).

Elshafei conducted a study in 1998 to examine the effect of using problem based learning on students' achievement and attitude. Sixteen Algebra II classes with eight teachers were randomly divided into two groups. Four teachers, with their eight classes, used problem-based methods to teach a unit while the other four teachers used the traditional approach to teach the same unit. From the data analysis, observations of participating students and teachers and their comments, Elshafei concluded that students and teachers favored problem-based method of teaching over the traditional method. She also concluded, "Students solving problems in groups preformed better and generated more plausible solutions than traditionally taught students, which is particularly significant for higher-level academic achievement" (p. vi).

In 1999, Dickerson conducted a study to compare the effect of five instructional approaches on students' problem solving achievement. Two of the approaches used the traditional method of teaching problem solving and three used problem posing. Dickerson defined problem posing as "all the activities students engaged in when they wrote their own problems in response to a teacher prompt or problem situation" (p. 17). Dickerson concluded, "All three problem posing treatment groups were successful in impacting student problem-solving achievement to a statistically significant degree" (p. 75). Dickerson's believes that "problem solving is one of the most important elements of the student's mathematical experience. It is a lifelong skill that will essential for successful and productive life experiences" (p. 72).

Jonassen (2000) said, "Most psychologists and educators regard problem solving as the most important learning outcome for life, Why? Virtually everyone, in their everyday and professional lives, regularly solves problems" (p. 63). He also indicated that the number of people who will be required to memorize information, then take tests, is very few. He criticized the type of problems students encounter in schools because such problems are not even close to the problems they will face in their real lives.

Using problem solving to teach will help students understand not only how, but also when and why a certain concept is being used. This is the desired learning with understanding. Kamaruddin, Kamariah, and Amin (2012) state that:

> If we emphasize the applications of school mathematics in a wide variety of problem solving situations two good things will happen; (1) the student will somehow learn the mathematics needed to solve these problems, and (2) seeing that mathematics is useful, he will be motivated to learn more mathematics. This viewpoint recommends that the mathematics curriculum be organized, not around its own internal structure, but around problem solving (p. 187).

According to Middleton and Spanias (1999) motivation is positively associated with achievement, they indicated that "achievement motivation in mathematics is highly influenced by instructional practices, and if appropriate practices are consistent over a long period of time, children can and do learn to enjoy and value mathematics" (p. 13).

Kercood conducted a study in 2000 to compare the effects of assisting students during a categorizing activity on their problem solving ability and understanding. Two groups of students were formed; one group had assistance while the other group had no help. Kercood concluded, "Students who were actively allowed to form categories on their own had higher accuracy in a subsequent problem solving task and took longer to complete this activity, than students who were earlier given a scheme of categorization by the examiner" (p. 88). Kercood believes that the active involvement of the no assistant group resulted in deeper understanding. This deep understanding and sense making required more time but resulted in being more accurate when solving problems. She indicated that "The better accuracy could be attributed to their active cognitive processing on the task" (p. 88).

One of the objectives of a study conducted by McIntosh (2011) was to evaluate the effect of using problem solving groups on students' ability. The researcher taught his 28 ninth grade students using problem solving methods. Students were given introductory and final surveys to reflect on their ability. McIntosh concluded:

> By giving students opportunities to develop particular problem solving skills, students will have to negotiate their way through a variety of thought processes, group interactions, mathematical concepts and misconceptions, problem solving strategies and awareness. With the support exemplified in this study, students can be helped to become more reflective of these processes; and by directing student attention to these processes, students can improve in all of these areas and in their metacognition – as the findings in this study suggest (p. 142).

Jonassen (2000) indicated that when learners focus on procedural knowledge they are focusing on the surface of the mathematical concept. This procedurally oriented method is not very challenging and is not enough for students to construct an expandable knowledge. If knowledge in not expandable it is very hard for a student, with such knowledge, to use it to solve real life problems. Using problem solving can help students construct an expandable knowledge.

Authentic problem solving. According to Lombardi (2007), "Learning researchers have distilled the essence of the authentic learning experience down to 10 design elements, providing educators with a useful checklist that can be adapted to any subject matter domain" (p. 3). According to Lombardi, these ten elements are: real-world relevance, ill-defined problem, sustained investigation, multiple sources and perspectives, collaboration, reflection, interdisciplinary perspective, integrated assessment, polished products, multiple interpretations and outcomes. Mathematics teachers need to keep in mind that if students view any concept as irrelevant to their lives, they will dismiss it after they finish its exam. To help keep any mathematical concept active and usable, using authentic problems can help link that topic to real life, which will encourage students to be active and engaged in their learning. When students are engaged and active in their learning they will construct their understanding of the concept which will improve their achievement in mathematics.

Students rarely see a connection between the mathematics they learn at school and their daily lives. If there is no connection between real life and the mathematics that they learn in school, it is very hard for them to value the benefits of what they learn. Bellamy and Mativo (2010) urge teachers to give students the opportunity to do problems in class, instead of listening and watching the teacher. They also encourage teachers to relate what they teach to real life because they believed that real life situations are crucial to students learning and understanding. Chang (2010) stated, "I find myself constantly looking for ways to convince students what they are learning is worthwhile and useful, and motivate difficult concepts with interesting applications that students can relate to" (p. 248). Chang's reasoning for doing so is that it helps students relate to the topic and it keeps them engaged. For students to be engaged they need to understand the value of what they are learning which will occur when what they are learning is related to their lives. Instead of spending the time convincing students that the concept is important and that they need it, let them explore its benefits through solving authentic problems.

DeBay conducted a study in 2013 to examine the effect of involving students in a real life mathematical urban planning projects using graphical representations on their mathematical understanding. 62 high school students were involved in this study. The results from a pre and post-survey, interviews and observations indicated a significant increase in students' understanding. DeBay concluded that "This indicates that as a result of students' involvement in the Urban Planning project, an overall understanding of using graphs in real-world situations has given students an increased understanding of solving questions that involve graphical representations" (p. 86). DeBay believed that students understanding was positively influenced by the way they viewed mathematics during the project "by having students interest increase through solving mathematical tasks that are rooted in meaningful, real-world contexts; students' belief that they can succeed in real-world mathematical tasks; and a shift in students' beliefs regarding the definition of 'doing mathematics'" (p. v).

Rigelman (2002) referred to problem solving as habits of mind that prepare students to real problems that they might encounter in their lives. To help students develop problem solving habits of mind, we need to make sure that they practice problem situations that are similar to what they will encounter in their real lives. When teachers use authentic problem solving, they will give students a chance to sample some of the problems they will encounter in real life. It is a great way to help students be involved and interested in their learning. Jonassen (2000) believed, "Students think harder and process material more deeply when they are interested and believe that they are able to solve the problem" (p. 71). Students will be interested if they can see that what they are doing is related to their lives, this will occur when teachers use authentic problem solving to teach mathematical concepts. Kirschner, Sweller, and Clark (2006) claims that a main property of problem solving approach is that "they challenge students to solve "authentic" problems or acquire complex knowledge in information-rich settings based on the assumption that having learners construct their own solutions leads to the most effective learning experience" (p. 76).

Choi conducted a study in 1995 to determine the effect of contextualization and complexity of situations on achievement. Students were randomly assigned to one of the following four treatment groups; simple /contextualized problems, complex/ contextualized problems, simple /decontextualized problems, and complex/decontextualized problems. Students were given a test after the treatment which included the four types of problems. Choi concluded, "Students who studied complex, contextualized problems preformed the best and students who studied complex decontextualized problems preformed the worst" (p. x). Choi also concluded that:

> real-life complexity promotes the transfer of knowledge because it helps students to develop higher-level thinking and select relevant information from the environment. In addition it helps students to develop problem-solving plans and to use resources productively. Therefore, when learning is situated in contexts which simulate real-world complexity, achievement should improve (p. 9).

Using problem solving gives the students the chance to deeply understand the concept because it allows students to construct their knowledge and understanding through their solution.

In a study conducted by Bottge (1999) to investigate the effect of contextualized problem solving on a group of average and below-average Algebra 1 students, he stated, "The results of this study support the practice of situating problems in a meaningful context for improving the mathematic problem-solving skills of low and average-achieving students"(p. 90). Bottge recommended moving away from the traditional system of teaching mathematics to contextualized problems to help prepare students solve problems that they will encounter in their work or their lives. Similarly, Cifarelli (1991) advises, "The idea that learning and cognition are situated suggests that learners build up their conceptual knowledge in the context of ongoing activity" (p. 2).

Authentic problems help students construct their conceptual understanding which will be reflected in their achievement in mathematics. Authentic problems help them deepen their understanding of the concept because they themselves constructed their understanding of the concept. This kind of understanding helps students expand their knowledge to solve new problems and improve their achievement. In 1998 Olson conducted a study to examine the effect of using realistic problem solving on students' performance. In this study and intervention, students were tested and their performance was scored using a scale of low, medium, high, and exceptional. At the beginning of the intervention Olson said, "42% of students scored low, 54% medium, and 4% high. None of the students gave an exceptional response at that time" (p. 25). The intervention lasted for three months in which 10 realistic problems were integrated into the curriculum. Students were evaluated multiple times during the study. It was noticeable that the number of students earning low scores had decreased and the number of students earning high scores had increased. On the final evaluation Olson indicated, "50% of the students had obtained a rating of high and 8% of the responses were considered exceptional" (p. 26). Olson concluded that using realistic problems helped to improve students' performance significantly, she said, "It appears that through clearly defined expectations and thorough analysis of various student responses, performance on realistic application problems have significantly improved" (p. 26).

The main goal of mathematics is to help students become flexible and fluent thinkers and to help them acquire the knowledge that can help them in their lives. Using authentic problem solving will help students value the mathematical concept they are working on and they will be interested in the information because it is relevant to their lives. As Choi (1995) advises, "Real-world problems are interwoven in everyday contexts. Therefore, when learning tasks are situated in everyday contexts which provide meaning and relevant experiences, the transfer of knowledge should increase" (p. 8).

Exponential functions

Functions are very essential in representing and solving real life situations. For students to use such a tool they must understand its benefits and experience its usage. Exponential functions are present in a lot of real life situations yet students are often deprived from exploring them in a real life sitting. Using authentic problem solving to teach exponential functions will help students understand when and how to use it to solve arising problems.

Using functions to represent and solve real life situations is a great tool for students to use. But students will not take advantage and value this great tool unless they understand and practice its usage in their lives. Most of the time functions in general are taught in a way that is totally disconnected from real life. In most Algebra books the exponential function unit is small and mainly focuses on procedural knowledge not on conceptual understanding. The problem with such approach is that the students will only master how to manipulate numbers and variables without having a clear understanding of the advantages and usage of functions. Stacey and MacGregor (1999) are not satisfied with such approach; they described it as "symbol pushing without reference to any real situation" (p. 25).

Real life applications include, understanding growth and decay situations such as; money, population, microorganisms, amount of medication in the blood, constant reduction of the value of goods, and carbon half-life are essential in helping prepare students to face future challenges. Simon (1997) indicated that "Mathematically modeling situations of growth can create a better understanding of growth in the world" (p. 3). In 2006, Hofacker conducted a study to evaluate the effect of using contemporary methods with the focus on modeling mathematical situations and problem solving on students understanding and success. One hundred and seventy college students were randomly divided into two groups; experimental group which consists of 95 students and control group which consists of 75 students. The experimental group used contemporary teaching method (CCA) with the focus on modeling, discovery-based learning, and contextualized problem solving. The control group used traditional methods (TCA) which uses lecturing techniques to teach students a list of procedures. Hofacker referenced the abbreviations used by Small (2004); "Small refers to these newly refocused college algebra courses as contemporary college algebra (CCA) courses. Courses based on a philosophy of preparing students for mainstream calculus will be referred to as traditional college algebra (TCA)" (p. 12). Both groups worked on linear and exponential functions. Three exams, ten questions in total, were administered during the experiment. From the data analysis Hofacker concluded that students in the experimental group score significantly higher over the ten questions combined. The mean for the experimental group was 44.42 while the mean for the control group was 30.85. Hofacker indicated that:

> Students from the contemporary cohort though exhibit a greater understanding with material set in the exponential content strand. They also show a greater ability than the TCA cohort to understand problems set within context. CCA students show more flexibility to exhibit techniques to deal with graphs and tables of information than the TCA students do (p. 175).

In most of the current mathematics curriculums, exponential functions take two to four sections that focus on procedural knowledge only. Students are given the formula for growth and decay, they practice how to plug the given numbers or information in the formula. At the end of the exercises a couple of word problems are presented but the numbers are very clearly identified, all the students have to do is plug in the information. No real-life problems are asked in which thinking is required beyond knowing which number goes where in the formula. Bradie

(1998) specified the reason for his dissatisfaction in the current approach of teaching exponential functions, "Coverage consists primarily of supplying the requisite formula for, say, the growth of a bacterial colony or the decay of some radioactive substance, without explaining or deriving the reasons that the exponential function appears in the formula" (p. 224). Students' will only apply the formula but will not be able to use their knowledge in any uprising problem in the future.

Donachy conducted a study in 2012 to evaluate the effect of using problem solving to teach exponential and logarithmic functions on students understanding and achievement. Two high school classes were part of the study one of which Donachy used problem solving to teach the unit while the other class he used traditional method of teaching. Using the results of the three assessments that Donachy gave to students, during and at the end of the unit, there was no significant difference in their scores. Donachy used his observation of his students during the unit to conclude, "The experimental group proved to be more engaged and enthusiastic in the learning of this unit. The application problems promoted more discussion and team work from the students as well" (p. 4). Donachy advised:

> Mathematics courses need to be made as relevant as possible to students' lives. If students can't relate to the material, and find some meaning in it, they will tune out and turn off during class time. When a student can infer meaning behind a concept, they will apply that concept leading to a better, longer retention of the information (p. 38).

For students to use exponential functions to solve problems they must understand its use and benefits. In other words they need to understand the concepts. To help students understand the concept, Bradie (1998) indicated that he gives a mathematical explanation why the exponential growth and decay formula is used. He indicated that if he does not do that he is putting his students in a disadvantage situation, he stated, "I have not fostered my students' ability to recognize other phenomena, either in mathematics or in other disciplines that use exponential functions" (p. 224). Students will fully understand the concept when they use it to solve real life problems.

Clark conducted a study in 2006 to examine the effect of using a conceptual real-life problem solving approach to teach functions on students' understanding and achievement. Four classes with 134 students participated in the study. Two classes are composed of a total of 67 students were the experimental group. They were taught using conceptual and collaboration approach. The other two classes were composed of 68 students were taught using procedurally based instructions. No significant different in achievement was noticed in the pre-test between the two groups. Clark indicated, "The post-test revealed that conceptual students had a better overall understanding of the function concept than procedural students" (p. 44). Students who were taught using authentic problem solving approach not only achieved better but also related functions to real life. Clark (2006) indicated:

> However, the students in both groups seemed to have different ideas of whether the functions were useful in their daily lives. The conceptual students defined a function as a relationship between variables that occur frequently in real-life situations; they thought of functions as real situations that would be useful in their daily lives. In contrast, the students in the control group tended to define a function as an equation or a symbolic rule. They stressed manipulative skills and memorized procedures. The procedural students could not see how functions would be useful outside of school (p. 52).

Simon (1997) advised, "Because exponential functions model critical real-life situations such as population growth and percent interest, it is imperative that students understand the nature of the rate of exponential growth" (p. 2). The desired depth of understanding of exponential functions will not be achieved by focusing on procedural knowledge. In a study conducted by Weber in (2002) to analyze students' understanding of exponential functions he concluded, "While all of the students in our study could compute exponents in simple cases, few students could reason about the process of exponentiation, thus, according to our theory, these students' knowledge of exponential and logarithmic functions will be limited" (p. 1). Additionally, Windsor (2011) indicated that one of the benefits of using problem solving to teach Algebra is that it might improve and develop students' mathematical thinking beyond the acquisition of procedural knowledge that they've been used to in most of the mathematical concepts. When students build their own knowledge they gain confidence in their mathematical abilities which will help them achieve better. Clark (2006) advised teachers to include real-life problem solving situations to help students connect the mathematics they learn in school to real life.

Effects of problem solving on students' attitudes

When students believe that they don't understand mathematics their effort and achievement will negatively be affected; they will not try hard and they are easily discouraged by a simple mistake. On the other hand, when students believe that they are good in mathematics their effort and achievement will be positively affected; they will not be discouraged if they do make a mistake. Students' attitude toward mathematics is influenced by their mathematics experience and the way they were taught. Using problem solving helps improve students' attitude which will contribute to improving their understanding and therefore their achievement. In 1999 Middleton and Spanias indicated:

> Teachers must teach knowledge and skills that are worth learning. In other words, students must understand that the mathematics instruction they receive is useful, both in immediate terms and in preparing them to learn more in the fields of mathematics and in areas in which mathematics can be applied (e.g., physics, business, etc.). Use of ill-structured, real-life problem situations in which the use of mathematics facilitates uncovering important and interesting knowledge promotes this understanding (p.12).

In 2007, Devens-Seligman conducted a study to examine the effect of problem solving on mathematics achievement and attitude. The experiment group consisted of seven teachers who used non-routine problem solving supplementary activities. Two different groups, each of which consists of 164 students, were used as the control group. A pre and a post-survey were completed by all students. Devens-Seligman indicated that "For the students in this study, both their attitudes and understanding of mathematics improved as a result of solving non-routine problems" (p. 113). Devens-Seligman concluded:

> Most students will find more challenging mathematics curriculum interesting and exciting, as opposed to frustrating or boring. Inherent in this idea is that the sense of accomplishment felt by students who successfully solve a difficult problem will provide motivation to study mathematics further. (p. 156).

Devens-Seligman (2007) advised, "Learning to persevere toward a solution is essential to understanding the process and remaining confident in one's ability to reach a solution. Today's children and tomorrow's adults need to be confident in stating that they really are good at mathematics."(p. 167)

Curtis conducted a study in 2006 to evaluate the effect of using problem solving, cooperative learning, and graphing calculators on students' attitude. Observations, student questionnaires, and attitude surveys were the method used to measure attitude changes. Curtis indicated that:

> The data analysis revealed that the four standards-based pedagogies, cooperative learning, discourse, problem solving, and graphing calculators, were used in the study. Although the attitude inventory did not show a large change in students' attitudes, the qualitative data points toward the results that most students found at least one of the teaching strategies impacted their attitude of being a student of mathematics with regards to confidence, motivation, value, and enjoyment (p. 155).

In 1994, Wade conducted a study to evaluate the effect of constructivist-based problem solving on students' attitude. A constructivist problem solving instructional program was used on the treatment group for a six week period. Observations and student interviews were used to measure any change in attitude. Wade concluded, "Qualitative data revealed that the constructivist-based mathematics problem solving instructional program caused a positive shift in students' attitude toward mathematics problem solving." (p. 119). She also concluded that such instructional program is very beneficial for students because it "not only enhances achievement but helps students persevere in mathematics problem solving and promotes metacognition by encouraging them to be reflective on their thinking" (p. 119).

Adamson conducted a study in 2005 to understand how does using real life problem solving and focusing on students' sense making and understanding affect students' attitude toward mathematics. Adamson stated, "I have long believed that connecting mathematics to real-world settings would help students overcome their negative feelings toward mathematics and serve to help students learn" (p. 73). The data collected revealed that before the study most of the students had a negative attitude toward mathematics. They do not see the relation between real life and the mathematics they are learning. They don't see the mathematics relevant or necessary to their education. They are taking the class because it is required. During the study some of the students, for the first time, reported that they really understood the mathematics that they are learning, enjoyed mathematics, and could see mathematics related to their real lives. Adamson indicated:

> This study revealed that the focus on sense-making had a profound impact on student attitudes toward mathematics. Students attributed their changing attitudes to the positive classroom environment that supported their sense making efforts, to the encouragement of classroom discourse, and to the nature of the mathematical modeling tasks (p. 219).

In 1998, Olson conducted a study to examine the effect of using realistic problem solving on students' performance and attitude. The students' responses to current attitude were very encouraging. Olson stated, "Seventy-five percent of the students possess positive or very positive attitudes towards math. Of the remaining 25%, only 10% classify their attitude as negative. None of the students feel very negative towards math" (p. 31). Regarding attitude change, Olson indicated that "one fourth of the students feel their attitudes have significantly improved and an additional 20% have noticed slight improvement. Although nearly 15% have reported a worsened attitude to some degree, this number is not as large as those whose attitudes have improved during the intervention" (p. 31).

To help improve students' understanding and achievement teachers need to pay attention to students' attitude. Deviating from the traditional methods of teaching will help improve students' attitude. Olson advised that "to reduce and prevent future negative attitudes towards mathematics, a variety of methods can be used. Integrating realistic application problems, writing, and cooperative learning are among the solutions" (p. 17). Curtis warned from the results of the negative attitude toward mathematics that "our society will see fewer mathematics majors and a workforce with minimal mathematical skills" (p. 163). She advised teachers to pay attention to students' attitude and be more informed about the benefits of non traditional methods in their classrooms.

Previously, researchers focused on the effect of using problem solving on students' understanding and sense-making as well as the effect of using problem solving on students' attitude and achievement. A number of studies compared students' achievement when using the traditional method to teach exponential functions to students' achievement when using problem solving and modeling to teach it. The focus in those studies was not on the combination of authentic problems and exponential functions and its effect on students' attitudes and achievement. The comparison of achievement and attitude between teaching exponential functions using authentic problem solving and teaching exponential functions using the traditional method was the focus of this study. While the quantitative data was used to give a general idea about the difference between the two teaching methods, the qualitative data clarified the difference between them using students' reflections, observations and interviews.

CHAPTER 3 METHODOLOGY

The drive of this study is to understand and evaluate the effect of using authentic problem solving to teach exponential functions on students' attitude and achievement. To evaluate this effect two groups of students was surveyed and tested before and after they were taught a unit about exponential functions "growth and decay." One group used group work and authentic problem solving methods, while the other group used a traditional lecture method. The purpose of this study is to answer the following questions:

1. How is students' achievement affected by the use of authentic, problem-based learning when teaching exponential functions?
2. How is students' attitude affected by the use of authentic, problem-based learning when teaching exponential functions?

Research design

This study utilizes a mixed method design. A mixed method design was defined by Johnson and Onwuegbuzie (2004) as "The class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study" (p. 17). Johnson and Onwuegbuzie also indicated, "Mixed methods research also is an attempt to legitimize the use of multiple approaches in answering research questions, rather than restricting or constraining researchers' choices" (p. 17). Caruth (2013) indicated, "Mixed methods research has become a valid alternative to either quantitative or qualitative research designs. It offers richer insights into the phenomenon being studied and allows the capture of information that might be missed by utilizing only one research design" (p. 112).

The goal of this research is to assess students' achievement and the change in their attitude. Using only quantitative data will not give the full picture of the effect of a teaching method on students' achievement and attitude. Creswell (2009) indicated, "There is more insight to be gained from the combination of both qualitative and quantitative research than either form by itself. Their combined use provides an expanded understanding of research problems" (p. 203). Caruth (2013) stated an advantage of using a mixed method design, "They point out that words, photos, and narratives can be used to add meaning to numbers while numbers can add precision to words, photos, and narratives" (p. 115). While the quantitative data provides numerical differences, the qualitative data provides detailed explanation of the data. Using a mixed design takes advantage of the benefits of both. Creswell (2008) stated that:

> Mixed method research is a good design to use if you seek to build on the strengths of both quantitative and qualitative data. Quantitative data, such as scores on instrument, yield specific numbers that can be statistically analyzed, can produce results to assess the frequency and magnitude of trends, and can provide useful information if you need to describe trends about a large number of people. However qualitative data, such as open-ended interviews that provide actual words of people in the study, offer many different perspectives on the study topic and provides a complex picture of the situation. (p. 552)

Quantitative data was collected from students' pre-and post-tests, quizzes, entry tickets, and from their pre-and post-attitude surveys. The qualitative data was collected from mathematical reflections, student' free response reflections, observations, and from the interviews.

The quantitative data would be useful in measuring the growth in their understanding before and after teaching the unit and the change in their attitude and to compare the change in achievement and attitude between the control and the experimental groups. Bordens and Abbott (1999) indicated, "Pretest-posttest designs are used to evaluate the effects of some change in the environment on subsequent performance. You might employ a pretest-posttest design to assess the effect of change in an educational environment" (p. 260); they also stated, "By using a pretest-posttest design, you compare levels of performance before the introduction of your change to levels of performance after the introduction of the change" (p. 260)

The qualitative data was useful to evaluate the change in student's achievement and attitude as the result of the teaching method they experienced. The interviews provided access to students' understanding, achievement and attitude that might not be accessed through the test. Students were given a student's reflection form to fill out in which they expressed their opinions and feelings about the teaching method they were exposed to and how it affected their view of mathematics. Caruth (2013) stated "Exploratory sequential, to first collect qualitative data to investigate a phenomenon and second gather quantitative data to explain the qualitative findings" (p. 114). Nenduradu's (2005) use of qualitative design helped him focus on the process rather than just the final outcome which helped clarify the phenomena. The use of mixed method design helped utilize both qualitative and quantitative data to add depth and precision to the study.

Participants

The students that participated in this study were in one of four Algebra 1 classes in two different suburban charter schools near a large city in the mid-western United States. Each school has around 700 students coming from a large mixture of different ethnic groups such as Indian, Hispanic, Chinese, Japanese, Middle Eastern, Romanian, and African American. They all have great parental involvement and support. Most parents provide lunch for their children, less than 50 students in each school receive free or reduced lunch. Students in the school take Northwest Evaluation Association (NWEA) Measures of Academic Progress (MAP) test to evaluate their overall understanding and achievement in mathematics. Students in each class, starting in third grade, are grouped into three levels according to their mathematics ability; the low group, the regular group, and the advanced group. Teachers are required to differentiate instructions according to the group to make sure they challenge each student.

The eighth grade students are grouped into three classes according to their mathematical MAP scores. The students in the low group learn eighth grade math while the students in the regular and advanced group learn Algebra 1. Most students in the two schools are hard working and are motivated to do well. The two classes in each school who participated in the study were the Algebra 1 classes. Their MAP scores exceeded their expected eighth grade scores which is why they were taking Algebra 1 in eighth grade. The parents of the students were very supportive by keeping track of their children's progress, making sure that they do their homework, and doing whatever it takes to help their children succeed.

Teachers

The teachers were eighth grade math teachers in each school. The first teacher (I will refer to as teacher A) has been teaching eighth grade mathematics class for five years. This teacher believes that effectiveness comes with consistency and routine. The second teacher (I will refer to as teacher B) has been teaching eighth grade mathematics for eight years. This teacher believes that cooperative learning combined with a gradual release of information is an effective teaching method. Teacher B taught the experimental group while teacher A taught the control group.

Setting

Four eighth grade Algebra 1 classes were used in this study; two classes were regular Algebra 1 and the other two classes were honors Algebra 1. The experimental and control group each consist of one regular class and one honors class. Each class period is 90 minutes daily. Since the two schools have a very similar student population in terms of their math abilities, one of the schools was the control and the other school was the experimental group. The control group with teacher A used the Big Ideas Algebra1 textbook (Larson &Boswell, 2014). The experimental group with teacher B used the exponential function unit using Connected Math Project 1 (CMP1). Wayne State's lesson plan format was used to write the lesson plans for each group. CMP has provided a plan for each lesson, so most of the suggestions and information they provided were incorporated into the lessons. Big ideas Algebra1 has an overview of the lesson therefore the lesson plans for the control group were created according to the researcher's experience and knowledge.

The following is a summary of what each group did in each of the ten days (see Table 1). For more details see each group's lesson plans in the Appendices A and B.

Table 1

Daily objectives of both groups

	Objectives for the control group	Objectives for the treatment group
12/2/15	• Use properties of exponents to multiply exponential expressions. • Use powers to model real-life problems.	• Gain an intuitive understanding of basic exponential growth patterns • Begin to recognize exponential patterns in tables, graphs, and equations • Understand the role of the growth factor in exponential relationships • Express a product of identical factors in both exponential form and standard form • Make a table from the graph of an exponential relationship
12/3/15	• Evaluate powers that have zero and negative exponents. • Graph exponential functions.	• Solve problems involving exponential growth • Write equations for exponential relationships represented by tables and graphs • Compare different exponential growth patterns and compare exponential and linear growth
12/4/14	• Use the division properties of exponents to	• Recognize exponential growth in verbal descriptions, tables, graphs, and equations

	evaluate powers and to simplify expressions.	• Determine and interpret the *y*-intercept (initial value) for an exponential relationship • Determine the growth factor based on a verbal description, table, graph, or equation for an exponential relationship
12/7/15	• Use the division property of exponents to solve real-life problems	• Write an equation for an exponential relationship from its graph • Solve problems involving exponents and exponential growth
12/8/15	• Use scientific notation to represent numbers. • Use scientific notation to describe real-life situations.	• Determine a non-whole number growth factor using information in a table or a graph • Determine the growth rate, or percent change • Use sample population data to write an equation to model population growth • Relate growth rates and growth factors • Review and extend understanding of percent • Understand the role of initial value (*y*-intercept) in compound growth.
12/11/15	• Write and use models for exponential growth. • Graph models for exponential growth.	• Understand the role of initial value (*y*-intercept) in compound growth • Use knowledge of exponential relationships to make tables and graphs and to write equations for exponential decay patterns
12/14/15	• Write and use models for exponential growth. • Graph models for exponential growth.	• Analyze and solve problems involving exponents and exponential decay • Recognize patterns of exponential decay in tables, graphs, and equations • Use information in a table or graph of an exponential relationship to write an equation
12/15/15	• Write and use models for exponential decay. • Graph models for exponential decay.	• Analyze an exponential decay relationship that is represented by an equation and use the equation to make a table and graph
12/16/15	• Write and use models for exponential decay. • Graph models for exponential decay.	• Examine patterns in the exponential and standard forms of powers of whole numbers • Use patterns in powers to estimate the ones digits for unknown powers • Use patterns in powers to develop rules for operating with exponents

12/17/15	• Write and use models for geometric sequence.	• Become skillful in operating with exponents in numeric and algebraic expressions • Describe how varying the values of *a* and *b* in the equation $y = a(bx)$ affects the graph of that equation

Every day the traditional group started the lesson by doing their bell work, which might be review problems or a small investigation to get them thinking about what will be taught that day. They checked homework and asked questions regarding their homework or regarding what they had learned the previous day. The teacher then collected their homework. Sometimes they received an entry ticket with one or two questions. The teacher introduced the new lesson, and the students took notes. At times they were given the opportunity to discuss a problem with a partner. At the end of the lesson the teacher reviewed the objective of the day and how they achieved it. At the end of class, the teacher gave the students their homework assignment.

The experimental group started their class by checking homework, which was collected by the teacher. The new lesson consisted of three parts:

A. Launch: the teacher launches the new problem by reviewing what the students did the previous day, connecting to prior knowledge, and encouraging students to investigate the task without giving away too much information.
B. Explore: the goal of the explore part is for students to explore a problem, which will enable them to analyze and generalize a concept or skill. Students attempted the task in small groups. This depends on the challenge and format of the problem.
C. Summarize: the purpose of this part of the lesson is to discuss, as a whole-group, students' discoveries of the lesson. During this portion of the lesson the teacher guided the students to reach the mathematical objectives of the problem and to connect their new understanding to prior mathematical knowledge. Student

conclusions and discoveries were shared and considered by their peers. Sometimes questions are posed for further exploration in future lessons. At the end the teacher summarized the finding and connected it to the objectives of the lesson then homework was given to the students.

The control group was taught using the traditional method; the teacher used power point presentation to teach the lessons (see control group lesson plans in Appendix B). In a ten day period the teacher spend four days teaching the properties of exponents and how to use each one to simplify exponents, one day on scientific notation, two days on exponential growth functions, two days on exponential decay functions, and one day on solving more problems on exponential functions. The teacher took the students step by step on how to use the rules to simplify exponents, graphing, writing and recognizing exponential growth and decay. For every concept introduced the teacher gave students multiple problems to practice and check their answers. It was teacher-lead teaching method, the students were asked to remain silent most of the time, take notes, listen and do the problems or ask questions. A couple of times during each lesson the teacher gave them the opportunity to compare their solution and their graph with the student next to them which took less than a minute each time. Students were given a worksheet, for homework, that has between 18 to 28 problems depending on the topic. The homework was straight forward, basically applying what they learned in class. The numbers in the problems were clearly labeled so students do not have to think a lot on what to do, it was just plug the numbers into the equation. The teacher checked the homework, went over the answers, and answered any questions the students had.

The treatment group used CMP1, which was based on group work and discussion. Students were assigned a problem to work on as a group. The teacher circulated between groups discussing their work and asking questions to make sure that they understood the concept correctly. In a ten day period, students worked on five investigations, each of which have its own objectives which mainly focus on writing and graphing exponential functions to represent given situations (see treatment group lesson plans in Appendix A). The teacher asked each group different questions, depending on their solution, to help them realize any misconception or misunderstanding in their work. If a group was having a difficult time solving the problem the teacher asked questions that guided them to the solution. The teachers' role was a guide, at no time direct instructions were provided. At the end of the class they discussed their findings as a whole class. The experimental group learning experience was a student-centered learning method using authentic problem solving. Students constructed their understanding through solving a series of realistic exponential growth and decay problems. Students were on task and engaged discussing their work all the time. The homework was an extension of what the students learned in class not just a repetition of the lessons learned in class. Each day student's had 3-5 problems with multiple questions in each problem. Each problem was a new real-life application of exponential functions. None of the problems had clearly identified numbers; instead students had to make sense of the problem in order to solve it. The teacher checked the homework daily, went over the answers, and answer questions students have.

The two methods use different ways for teaching and introducing the concept. To compare the way the two methods differ in teaching the chapter, I included a preview of the first lesson for both groups.

The control group first lesson

The objectives of the lesson are;

- Use properties of exponents to multiply exponential expressions.

- Use powers to model real-life problems.

As an introduction to using the multiplication property of exponents students started with an investigating activity. In this activity students had to fill in a provided table by multiplying numbers in exponential form. The teacher role was to explain the lesson and give examples while the students' role was to take notes, ask questions or answer questions. The lesson was divided into three sections. The first section was the product of power property, the second section was the power of power property, and the last section was the power of product property. For the first two sections the teacher started by giving the main rule, the new vocabulary, and an example to clarify the rule. In the third section the teacher explained how to use the first two properties to come up with the third property. In each section the teacher guided the students through four examples then gave them four similar questions to answer on their own. The last section included an extra two examples. One showed students how to use all learned properties together to simplify exponential expressions, and the other was a real world problem in which students had to use one of the properties. The lesson ended by giving students their homework. The book had 59 problems as exercises for the lesson. Fifty-one of them were direct applications of the properties, the last seven were labeled problem solving, and the last problem was labeled challenging.

The experimental group first lesson

The objectives of the lesson are:

- Gain an intuitive understanding of basic exponential growth patterns
- Begin to recognize exponential patterns in tables, graphs, and equations
- Understand the role of the growth factor in exponential relationships
- Express a product of identical factors in both exponential form and standard form

Students start with hands on activity that will help them review the properties of exponents. They investigated the growth in the number of ballots created by repeatedly cutting a piece of paper in half. They made a table and a graph that relates the number of cuts and the number of ballots. Students were asked to find a pattern between the number of cuts and the number of ballots to help them predict how many ballots will result in 20, 40, and 50 cuts. Their homework was six questions one of them was cutting a piece of paper into three pieces and the rest was to review converting between standard form and exponential format and to review the product and division properties of exponents.

Curriculum

The control group used Algebra 1 by Big Ideas (2014) and supplemented using Algebra 1 Holt, the experimental group used CMP1 exponential functions unit Growing Growing Growing. The reason for choosing that specific control group curriculum is that it is the book that's been used in the participating school. In addition most of the Algebra1 books address exponential functions in the same way. Since Holt and McDougal Littell were used in the school before Big Ideas, teachers had all the teaching material to supplement the current curriculum. The experimental group curriculum choice was because this was one of a few curriculums that teach exponential functions using problem solving.

Data collection

To evaluate and measure students' learning before and after the unit, students in both groups were given pre-and post-test, daily entry ticket, quizzes, homework, and four mathematical reflections writings. Parents were given the choice of having their child in the study. If a parent did not want their child in study no data was collected from that student. The post-test was given to each class separately after they finished the unit.

To measure any changes in attitude and in how they felt about the way mathematics is being taught, students were given a pre-and a post-survey with questions regarding their attitude toward mathematics and how relevant mathematics is, in their view, to real life. The survey was divided into two sections; one contained positive statements and the other contained negative statements to help capture students attitude and view of mathematics. The scale was flipped between the positive and negative statements to get an accurate measure. The qualitative data was collected by having the students, in both groups, fill out a free response feedback form in which they expressed their opinions and feelings about the teaching method they were exposed to and how it affected their view of mathematics. During class, while the teacher was teaching, the researcher observed and gathered information focusing on students' behavior, comments, engagement, and facial expressions. Their comments for each question were grouped into two groups, the positive and the negative. In addition 20% of each group was interviewed. Table 2 includes more information about all the data collection methods.

Table 2.

Data collection methods

Data collection type	Overview
Pre-and post-test	This test was given to students before and after the unit. It helped measure improvement in their understanding and knowledge. The test includes problems to measure students' conceptual knowledge and problems to measure students' procedural understanding. The first ten questions were taken from McDougal Littell Unit test and the rest of the questions from CMP test generator. Expert panel was formed to provide validity for this test. Three teachers who teach the same class from different schools and faculty members at the university provided feedback from this test. Their suggestions were incorporated into the test and their cumulative opinion was that the test was a good measurement for students understanding for the unit. (See Appendix C for the pre-and post-test.)
Pre-and post-survey	This survey contains a number of questions to help measure students' attitude toward mathematics and its relevance to their lives. Students will answer this questionnaire twice; once before the unit and once after the unit. This will help measure any changes in students' attitude (see Appendix D for the attitude

survey). The attitude survey is based on three studies each of which used an attitude survey. Wade (1994) developed an attitude survey to measure the change in students' attitudes after six weeks of constructivist-based problem solving instructions. Olson (1998) used an attitude survey to measure the change in students' attitudes after integrating realistic problem solving into their mathematics curriculum for a period of five months. Curtis (2006) developed a survey to measure students' attitudes after using problem base learning treatment. Two administrators and three teachers were given the survey questions in order to evaluate the survey in terms of how well it measured attitude. Their feedback was to mix the positive and negative questions, and to ask students to read carefully before answering. These suggestions were followed.

Homework	Students in the treatment group and in the control group completed a daily homework which helped students practice the skills and concepts they learned that day. The homework was checked as a class by posting the correct answers then answering any questions students have regarding the homework. The homework was collected so that the researcher can check for accuracy and how the students arrived at their answers. Homework is a good tool to help discover any misconceptions students might acquire
Observation	The teachers were observed by the researcher using the school's observation protocol. The observation focuses on the areas where the teacher is doing a great job and the areas where the teacher needs to improve in. The main focus is on students' understanding and engagement as well as the teaching style (see Appendix I for the Observation Protocol).
Mathematical reflection	Students reflected on their learning and understanding by answering questions on their thinking process when they encountered the problems in exponential growth and decay and in simplifying exponents. They did five mathematical reflects during the unit (see Appendix H for the Mathematical Reflections).
Quizzes	Students had two check points to measure students' understanding of multiple objectives. This helped the teacher and the students see their weak areas and work on improving it (see Appendix J for the Quizzes).
Interviews	I interviewed 20% of the students from each groups. The main focus of the interviews was to get their feedback on the teaching method they were exposed to. It helped shed more light on their attitude while they are learning using the specific teaching method. The feedback of three teachers was considered to help validate the questions (see Appendix K for the Interview questions).
Entry tickets	Each day before the lesson, students had a fast check point of the previous material by answering three to four questions (see Appendices E and F for Entry Tickets).

Reflection Sheet	Students wrote about their learning experience and if they noticed any changes in their attitude by answering six questions (see Appendix G for the Reflection Sheet).

Data Analysis

The independent t-test, the dependent t-test, and Wilcoxon Mann-Whitney test were used to analyze the quantitative data. The results from the t-test and the Wilcoxon Mann-Whitney test led to similar results. Elmore & Woehlke (1997) stated, "The simplest definition of independent samples is that the groups being studied are mutually exclusive" (p. 161). Elmore & Woehlke (1997) also indicated that dependent samples are when "subjects are measured at time 1 and then the same subjects are measured for time 2" (p. 161). In 2010, Ho conducted a study to determine the impact of different levels of task difficulty and expertise on self-efficacy judgments. Ho used the independent sample t-test in her research. Ho stated, "An independent samples t-tests were conducted to determine if there were any differences between the low level and high level of expertise groups as well as the different task difficulty groups." (p. 54). The independent and the dependent t-tests were used to measure any change in students' attitude.

The independent t-test was used to measure the difference in achievement between the two groups in the pre-test and post-test as well as the difference in attitude between the two groups in the pre-and the post-survey. It was used to measure the difference in the improvement in achievement and attitude from the beginning to the end of the experiment. Using the independent t-test shed some light on the difference between the two groups before the experiment and whether the difference between them got bigger or smaller after the treatment.

The dependent t-test was used to evaluate the change in students' achievement and attitude after teaching the unit. This test was used to compare the pre-test and the post-test for achievement and attitude changes of each group separately to understand the effect of the used teaching method on their achievement and attitude.

To analyze the qualitative data, Creswell (2009) advised to start with the following three steps:

1. Organize and prepare the data for analysis.
2. Read and look at all the data. This first step provides a general sense of the information and an opportunity to reflect on its overall meaning.
3. Start coding all of the data. Coding is the process of organizing the data (p. 197).

Once I organized all the qualitative data that I collected, I read it then organized it into two groups; one for the experimental group and one for the control group. Then I use inductive analysis to interpret the data. In 2006, Thomas stated, “Inductive analysis refers to approaches that primarily use detailed readings of raw data to derive concepts, themes, or a model through interpretations made from the raw data by an evaluator or researcher” (p. 238). He also indicated, “The primary purpose of the inductive approach is to allow research findings to emerge from the frequent, dominant, or significant themes inherent in raw data, without the restraints imposed by structured methodologies” (p. 238). Strauss and Corbin (1998) indicated that in the inductive method the researcher starts by reading the raw data which leads him to form a theory regarding the data.

The qualitative data was collected from the interviews, the students’ reflections, and the classroom observations. The data was organized into two groups: the positives and the negatives of each teaching method. The qualitative data was used to clarify the difference in attitude between the two groups and the reason for that difference. The qualitative and quantitative data made it clear what is the effect of the different teaching method on students' attitude and achievement.

Lesson learned from the pilot study

I conducted a pilot study in April 2014 to help develop and strengthen the current study. I used a matched pair design to divide my students into two similar groups. One group was taught exponential functions using the traditional method and the other group used authentic problem solving approach to learn exponential functions. I taught the treatment group while another teacher taught the control group. I used mixed methods to collect my data. Quantitative data was collected from pre-and post-test to measure achievement and pre-and post-attitude survey. The qualitative data was collected from classroom observation and a questionnaire that the students had to fill.

In the pilot study, I taught the treatment group while another teacher taught the control group. This allowed room for bias considering I am writing the study and have more experience than the other teacher. I did not teach any group during the actual study; Instead, I was observing two teachers each in their own classroom. One teacher taught the treatment group while the other teacher taught the control group. Both are about the same level of experience in teaching. I created the lesson plans for both groups. The treatment group teacher was trained for a couple of days on how to teach using authentic problem solving using CMP, while the other teacher did not need any training. Since I have two groups in two different schools, sometimes I observed one teacher each day, other times I observed one of them for the first half of the day then went to the other teacher for the second half.

In the pilot study students reviewed exponents at the beginning of the year so when it came to teaching the unit in the control group they already knew the entire unit with the exception of the last two sections. This controlled the amount of time they spent on the unit, which was just four days. The treatment group had no lessons on simplifying exponents in the unit which resulted in no reduction of the required time for the unit, their unit took ten days. I changed that for the actual study, the students did not review exponents at the beginning of the year but they worked on it as part of their unit. Both groups took ten days to finish the unit.

Data collection was not enough to give a clear picture in the change of achievement and attitude. Data was collected only from the pre-and the post-test and observation. In the actual study multiple data sets was collected: for achievement pre-and post-test, daily entry tickets, quizzes, mathematical reflections, interviews, and observations. For the change in their attitude I used pre-and post-surveys, interviews, observation, and students' reflection worksheets.

Attitude survey in the pilot study was divided into two groups each of which have two sections, one section included all positive statements and the other section included all negative statements. In this study the two sections the questions were mixed, students were advised to read the questions well before they answer them.

Validity

Hugh, Parry and Crossley (1950) defined validity as "The ability of a test to predict performance" (p. 61). The procedures that the researcher uses to measure validity were indicated by Creswell and Miller (2000) as "procedures for validity include those strategies used by researchers to establish the credibility of their study (p. 125). For my particular research I have to determine the validity of the instruments that I used to collect quantitative and the qualitative data. Creswell and Miller (2000) listed five methods that researchers use to ensure the validity of their results; they are "member checking, triangulation, thick description, peer reviews, and external audits. Researchers engage in one or more of these procedures and report results in their investigations" (p. 124).

In this research, I used both data triangulation and expert panels in an effort to have a better representation of the data. Creswell and Miller (2000) defined triangulation as "a validity procedure where researchers search for convergence among multiple and different sources of information to form themes or categories in a study" (p. 126).

Triangulation was used for the collection of both qualitative and quantitative data. To measure achievement, multiple ways to collect data were used; pre-and post-tests, quizzes, entry tickets, and mathematical reflections. All of these resources give a more accurate measure of the change in students' achievement. These multiple resources gave a more accurate measure of the difference in achievement between the experimental group and the control group. For attitude, a pre-and a post-survey was utilized to measure the numerical change in attitude and to measure the difference in attitude between the two groups. Triangulation was used when I collected multiple forms of information; this includes formal observations, students' reflections, mathematical reflections, and interviews.

Expert panels were utilized for the qualitative and quantitative data. I used expert opinions to make sure that the used tests, quizzes, and entry tickets measure the intended content area. Three teachers who teach the same class from different schools, as well as faculty members at the university provided feedback for the measurement. Their suggestions were then taken into consideration and the test was adjusted to reflect their feedback.

The attitude survey was created based on the literature. It was validated by taking the feedback of two administrators and three teachers who evaluated the survey questions in terms of how well it measured attitude. The survey was adjusted to reflect their feedback. For the qualitative data; three teachers were asked to review the interview questions and students' reflection questions to make sure that the questions were not misleading. Their feedback was considered and the questions were adjusted to reflect their cumulative suggestions.

CHAPTER 4 RESULTS

My research questions are:

1. How is students' achievement affected by the use of authentic, problem-based learning when teaching exponential functions?
2. How is students' attitude affected by the use of authentic, problem-based learning when teaching exponential functions?

I divided my data into two sections. One section focused on achievement, which deals with students understanding and ability to solve and explain their solution, and the other section focused on students' attitude toward mathematics in general as well as their feelings toward problem solving during and after the unit. I started my analysis by reading multiple sources of data multiple times. Furber in 2010 advised that:

> The first stage of data analysis involves the researchers immersing themselves in the data in order to be completely familiar with it, and develop an overview of the main ideas in the data. This means reading and re-reading the transcripts in order to gain this level of understanding (p. 98).

The first round of reading helped me have an overview of the information in the data. During the second round of reading, I was focusing on how to group and divide the information in a way that makes sense. In 2002, Mason indicated that:

> At first sight, this kind of sorting and ordering of data seems an entirely practical task which can be done according to certain technical indexing and cataloguing conventions. Viewed in this way, it seems that once the data are sorted and ordered, the researcher will start to be able to make some interpretive sense of them, and to build their explanations and arguments (p. 148).

The data from this study had two main focuses; the first was students' mathematical understanding and achievements, and the second was students' attitudes towards mathematics. To analyze each of them, I used cross-sectional data indexing. Mason in 2002 stated:

> the logic of cross-sectional data indexing is that you devise the same set of indexing categories for use, cross-sectionally, across the whole of your data set. In other words, you are using the same lens to explore patterns and themes which occur across your data (p. 165).

I analyzed the achievement data question by question as well as the total score for each student. My focus was on how many students answered each question correctly and categories of the ways they solved each problem. The attitude data was also analyzed question by question. I grouped their answers into positive, negative, or neither positive nor negative categories. Huberman and Miles (1994) agreed with Mishler (1990) that "qualitative studies ultimately aim to describe and explain (at some level) a pattern of relationships, which can be done only with a set of conceptually specified analytic categories" (p. 431).

Students' mathematical achievement

Student achievement was analyzed using results from pre-and post-tests, quizzes, entry tickets, mathematical reflections, observations, and interviews. Each of these was analyzed question by question with a focus on the number of students who answered each question correctly, as well as a focus on the way each question was answered. Both qualitative and quantitative data showed that the students in the experimental group performed better than the control group. Before the unit, the control group scored higher than the experimental group in the pre-test. After the unit, the students in the experimental group scored higher than the students in the control group in the post-test. The means of each of the entry tickets and quizzes for the experimental group were higher that most of the means of each of the entry tickets and quizzes for the control group. The results of the analysis of the mathematical reflections showed that the percentage of students who answered the questions correctly was higher for the experimental group than it was for the control group. The analysis of the interviews with students revealed that the students in the experimental group reflected on their thinking and understanding more that students in the control group.

Pre-and post-test results. To measure students' achievement for the unit, students took a pre-test before the unit and a post-test after they finished the unit. I analyzed the data question by question to determine how many students answered each question correctly before and after the unit.

To determine if there was any significant difference between the two groups, the independent two sample t-test was calculated for the mean scores of each question in the pre-test, the mean scores of each question in the post-test, the mean scores of the pre-test total scores for each student, and the mean scores of the post-test total scores for each student. The Wilcoxon Mann-Whitney test was used but it gave the same results as the t-test.

It was found that the control group started with a higher achievement level than the experimental group (see Table 3). The mean scores of the pre-test total score were significantly different in favor of the control group. For the question by question results; there was no significant difference between the two groups in five questions. The groups were significantly different in two questions, both of which were in favor of the control group. No one in the two groups answered questions seven and eight correctly which resulted in no test results for these two questions.

Table 3

Pre-test comparison between control group and experimental group using the t-test.

	Control group		Experimental group			
Question Number	Mean	SD	Mean	SD	t-test	*p*
Question 1(7 parts)	0.78	1.294	0.8	1.34	-0.52	0.958
Question 2(3 parts)	0.54	0.84	0.27	0.624	1.635	0.106
Question 3(2 parts)	0.2	0.511	0.11	0.321	0.887	0.378
Question 4(2 parts)	0.2	0.156	0.9	0.362	-1.113	0.270
Question 5(2 parts)	1.15	0.823	0.48	0.549	4.374	0.00*
Question 6(1 part)	0.12	0.331	0.02	0.151	1.756	0.085
Question 7(3 parts)	No one answered this question right. No test results					
Question 8 (2 parts)	No one answered this question right. No test results					
Question 9 (5 parts)	0.66	1.063	0.07	0.334	3.402	0.001*
Total score for the pre-test	3.46	2.637	1.82	1.769	3.353	0.001*

* $p < 0.05$.
SD: Standard Deviation. The test has nine main questions. Each question consists of multiple parts. Each part is worth one point. The maximum possible score is 27.

The post-test was identical to the pre-test and was given after they finished the unit. There was no significant difference between the two groups in three questions (see Table 4). There was a significant difference in favor of the experimental group in the remaining six questions and in the total score in the post-test. There was no question in which the control group scored significantly higher than the experimental.

Table 4

Post-test comparison between control group and experimental group using the t-test.

	Control group		Experimental group			
Question Number	Mean	SD	Mean	SD	t-test	*p*
Question 1(7 parts)	4.41	2.133	5.61	1.418	-3.030	0.003*
Question 2(3 parts)	2.51	0.952	2.52	0.976	-.050	0.960
Question 3(2 parts)	1.54	0.636	1.48	.549	.461	0.646
Question 4(2 parts)	1.17	0.919	1.75	.576	-3.453	0.001*
Question 5(2 parts)	1.54	1.845	1.52	.664	0.047	0.963
Question 6(1 part)	0.22	0.419	0.82	.390	-6.821	0.000*
Question 7(3 parts)	1.98	1.172	2.55	.875	-2.525	0.014*
Question 8 (2 parts)	0.78	0.962	1.41	0.897	-3.109	0.003*
Question 9 (5 parts)	2.45	1.782	3.82	1.299	-3.987	0.000*
Total score for the post-test	16.55	6.551	21.48	4.781	-3.904	0.000*

* $p < 0.05$.

SD: Standard Deviation. The test has nine main questions. Each question consists of multiple parts. Each part is worth one point. The maximum possible score is 27.

To compare the increase in students' achievements from the beginning of the study until the end, the total score was calculated for every student in the pre-test as well as the total score in the post-test. The pre-test total was subtracted from the post-test total for every student. The two-sample t-tests result indicated that there was a significant difference in the achievement in favor of the experimental group with a t-value of -5.625 and a p-value of 0.00. The mean growth of the experimental group was 19.66, while the mean growth of the control group was 13.07. This provides evidence that the teaching method helped increase student achievement more effectively. The control group started off at an advantage in their pre-test scores, but this advantage diminished as the students in the experimental group were engaged in the learning process while constructing their understanding of the concepts. This shows that this type of engagement helps students excel in their learning.

Quizzes and entry tickets results. When students finished a lesson, they were assigned some problems for homework. Entry tickets were given the following day after students checked their homework (see Appendix C and D). The purpose of the entry tickets was to measure how much information students retained from the lesson. Each group had questions based on what they learned. The control group curriculum was focused more on procedural knowledge, so their entry ticket questions were similar to their homework. The questions in their quizzes were designed to measure their learning which was focused on procedural knowledge. The experimental group curriculum was focused on conceptual understanding. Their homework was an extension to what they did in the classroom, so their entry tickets were similar to their homework. The quizzes for this group were also similar to what they learned; the focus was on conceptual knowledge.

The mean of each entry ticket as well as the mean of the two quizzes for both groups were calculated to help observe the achievement of two groups. The means were grouped into intervals to help give a general idea about the achievement of each group. It was noticed that 92% of the mean scores of the experimental group were higher than 70, while 40% of the control group's mean scores were higher than 70 (see Table 5).

Table 5

The mean scores of the entry tickets and quizzes

	Experimental group	Control group
Mean ≥90	8%	0%
80≤ mean<90	58%	30%
70≤ mean<80	25%	10%
60≤ mean<70	8%	30%
50≤ mean<60	0%	30%

Each assessment consisted of a couple of questions; each one is worth one point. The scores range from zero to 100.

Every entry ticket was graded out of 100. To help analyze the grades of the two groups, the mean and the median of each assessment was calculated. The medians were included because the mean is easily influenced by outliers (see Table 6). The experimental group had seven assessments in which the median was 100 which indicate that at least half of the students scored a 100% in those assessments. The control group had two assessments in which the median was 100. While the median in the experimental group was greater than 66 in all assessments, the control group had five assessments in which the median was less than 66. This indicates that at least half the students in the control scored less than 66 in five assessments.

Table 6

Quizzes and entry tickets scores for the two groups.

	Experimental			Control		
	Mean	Median	SD	Mean	Median	SD
Entry ticket 1	88.62	83.33	11.401	67.27	66.67	21.106
Entry ticket 2	70.96	75	10.076	51.35	44.44	25.972
Entry ticket 3	85.88	100	27.647	62.16	57.14	24.097
Entry ticket 4	74.26	75	24.989	54.73	50	30.950
Entry ticket 5	79.41	85.71	20.564	83.78	100	23.069
Entry ticket 6	71.24	66.67	32.030	83.78	100	23.729
Entry ticket 7	88.24	100	28.361	69.59	75	69.59
Entry ticket 8	88.97	100	21.489	58.11	50	44.897
Entry ticket 9	82.35	100	26.493	Did not take entry ticket 9		
Entry ticket 10	100.00	100	13.599	Did not take entry ticket 10		
Quiz 1	88.24	100	13.987	80.07	87.50	17.544
Quiz 2	91.42	100	13.987	73.51	80	25.410

SD: Standard Deviation. Scores ranges from zero to 100

Mathematical reflections. Students in both groups answered the same four mathematical reflections, each of which consisted of a number of questions regarding the material they learned. To analyze it, I focused on the number of students who answered the questions correctly. The percent of students who answered correctly in the experimental group was higher than the control group in seven questions, while the control group's percentage was higher in six questions. Table 7 displays the questions and the percentage of students who got the respective question correct from each group.

Procedural knowledge questions were answered with a higher percentage of correct responses by the control group. Most of the work that was done by the control group was focused on procedures by applying a given rule or formula. When students were asked to demonstrate conceptual understanding, to explain their reasoning, or to compare and contrast ideas, the experimental group had a higher percentage of correct answers. Using a student-centered curriculum helped students construct their conceptual understanding in a way that allowed them to answer in-depth questions, whereas using the traditional curriculum helped students to have a strong procedural understanding of the material.

Students answered four mathematical reflections. Table 7 displays a summary of their percentages then a question by question analysis for each of the mathematical reflections follows

Table 7

Percentages of correct answers in the mathematical reflection questions

Assessment	Question	Control Group	Experimental Group
Mathematical Reflection 1	1a. Describe some of the rules for operating with exponents	73%	71%
	1b. What is scientific notation? What are its practical applications?	51%	77%
	2. Compare exponential and linear functions.	71%	87%
Mathematical Reflection 2	1. Describe an exponential growth pattern. Include key properties such as growth factors.	7%	87%
Mathematical Reflection 3	1. How can you use a table, a graph, and an equation to find the y-intercept and the growth factor of the function?	22%	25%
	2. How can you use the y-intercept and growth factor to write an equation that represents an exponential function? Explain.	39%	48%
	3. How would you change your answers to questions 1&2 for a linear equation?	34%	32%
Mathematical Reflection 4	Suppose you are given the initial value for a population and the yearly growth rate. 1a. How can you determine the population several years from now	44%	34%
	1b. How is growth rate related to growth factor?	24%	16%
	1c. How can you use this information to write an equation that models the situation?	54%	41%
	Suppose you know the initial value for a population and the yearly growth factor. 2a. How can you determine the population several years from now?	39%	45%
	2b. How can you determine the yearly growth rate?	24%	14%
	3. Suppose you know the equation that represents the exponential function relating the population p and the number of year's n. How can you determine the doubling time for the population?	12%	34%

Mathematical reflection 1. This reflection consists of two questions:

Question 1: Describe some of the rules for operating with exponents, the meaning of scientific notation and its practical applications.

The experimental group. Of the 44 students 71% wrote and explained all the rules for operating with exponents. Eleven students (25%) extended their answers by providing examples to explain each rule (see Figure 1). Ten out of 44 students did not manage to write the rules for simplifying exponents.

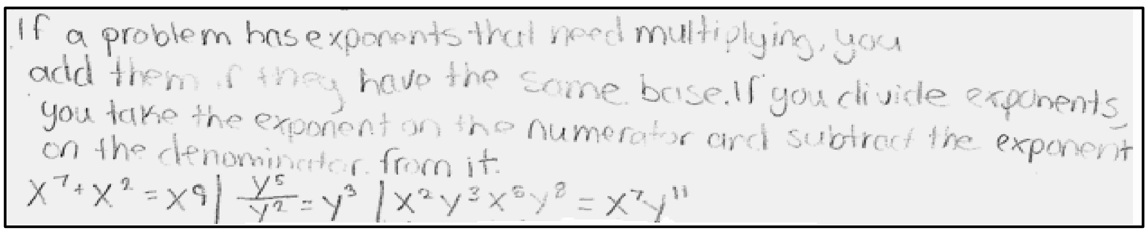

If a problem has exponents that need multiplying, you add them if they have the same base. If you divide exponents you take the exponent on the numerator and subtract the exponent on the denominator from it.
$x^7 + x^2 = x^9$ | $\frac{y^5}{y^2} = y^3$ | $x^2 y^3 x^5 y^8 = x^7 y^{11}$

Figure 1. Simplifying exponents rules from a student in the experimental group.

In the scientific notation part, 77% of the 44 students were adept at explaining the meaning of scientific notation; three of them managed to present real-life applications of scientific notation such as finding the mass of a planet (see Figure 2). Ten of the students did not provide an adequate answer; some of them only wrote that it was a way to write big numbers in a short way.

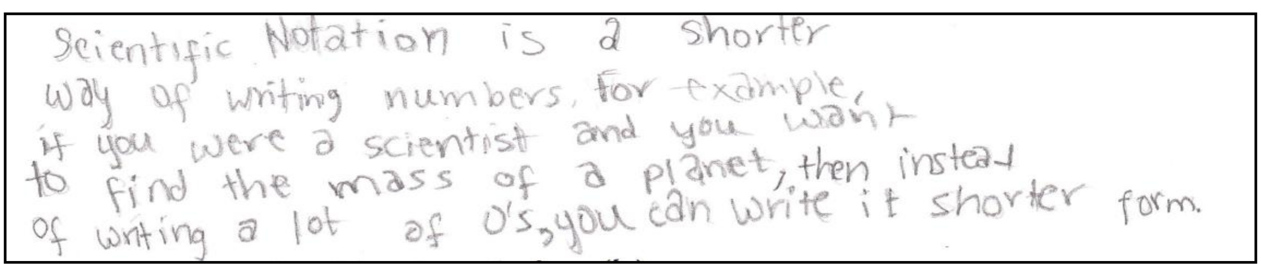

Scientific Notation is a shorter way of writing numbers, for example, if you were a scientist and you want to find the mass of a planet, then instead of writing a lot of 0's, you can write it shorter form.

Figure 2. The meaning of scientific notation as written by a student in the experimental group.

The control group. Of the 41 students 73% showed proficiency in explaining the rules of simplifying exponents some of them wrote the rules then added examples to clarify their answer. One student wrote the name of the property and gave examples to help explain the rule (see Figure 3). Some students mentioned that you can only simplify if the bases are the same. Eleven students did not manage to write the rules correctly or wrote one rule.

You have 4 rules/Property
Power of Power = $(\frac{1}{2})^3$ You raise both the numerater & dinomitor
Power of Multipaction = $10^3 \cdot 10^4$ You add exponet if the bases are the sam
Power of dividing = $\frac{4^7}{4^5}$ You subtract exponets if they have same bas
? last one forgot name = $(\frac{1}{2}^4)^3$ You multiply exponets & give everthing inside the paratheses the exponet on the outside

Figure 3. Simplifying exponents rules from a student in the control group.

In the scientific notation part; 51% of the students explained the meaning of scientific notation, two of them gave a real-life application of scientific notation, both of whom said it could be used to measure the distance between planets (see Figure 4).Of 41students 49% did not manage to explain the meaning of scientific notation.

Scientific notation is close to another way of simplifying something. For example if you rewrite 78,000 in scientific notation you would write it as $7.8 \cdot 10^4$. Something you might use it for is comparing distances between different planets and the sun.

Figure 4. The meaning of scientific notation as written by a student in the control group

Question 2: Compare exponential and linear functions. Both groups learned linear functions at the end of September - beginning of October.

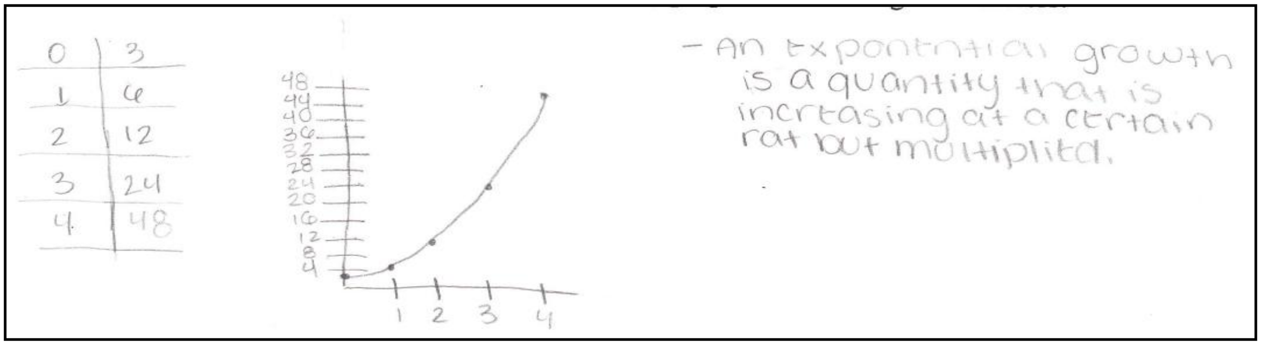
The experimental group. Of the 44 students, 87% explained the difference between linear and exponential functions in terms of how each function increases or decreases and in terms of the shape of the graph. On the third day of the unit they were asked to compare a linear function with an exponential function. They were also asked to write equations to situations that represented linear growth. Five students (11%) did not manage to sufficiently explain the difference between the two functions.

The control group. Of 41 students, 71% students adequately explained the difference between linear and exponential functions in terms of the shape of the graph, its equations, and how each one increases or decreases. Throughout the unit, students were not asked to graph a linear function or compare exponential to linear. The conversations students were having after submitting the worksheets made evident that they were struggling to recall the properties of linear functions. Twelve students (29%) did not manage to explain the difference between the two functions. They wrote some information down, but their thoughts were neither organized nor clear.

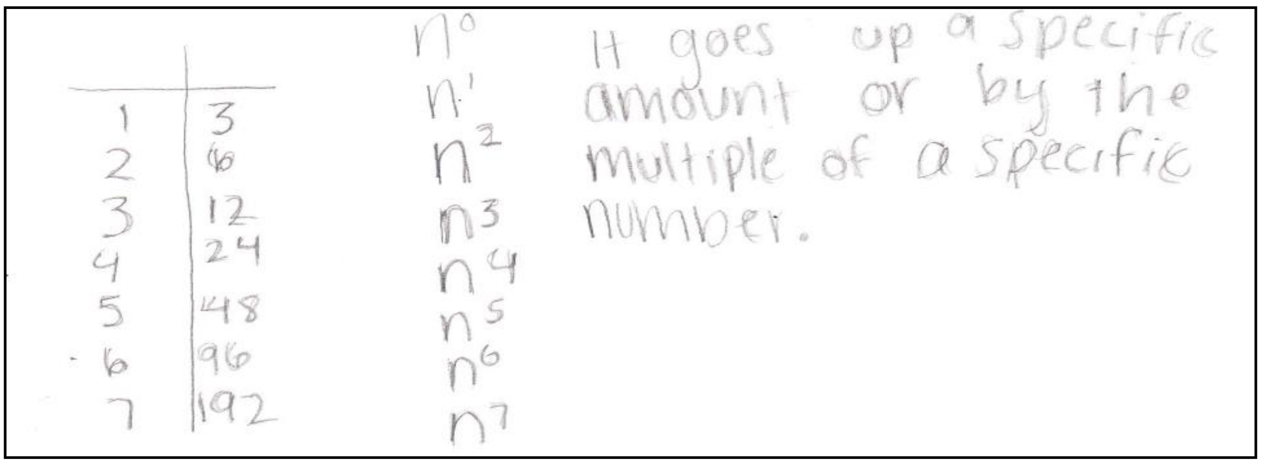
During the unit, the control group never had a chance to compare and contrast between linear and exponential functions. The curriculum did not ask any questions to help them recall what they knew about linear functions. On the other hand, the experimental group started comparing linear and exponential from the third day of the study. They had multiple chances to recall what they knew about linear functions and compare it to exponential functions.

Mathematical reflection 2. Students were asked to describe an exponential growth pattern while including key properties.

The experimental group. Of the 44 students 87% gave correct examples of exponential growth patterns. Only five students (13%) did not manage to give adequate examples. Students gave examples such as: 2^n, 3^{n-1}, 4^x, when you are doubling or tripling a certain amount, and a plant double its height each month. Some students used graphs and tables to indicate key properties (see Figure 5a and b). No students made mention of anything regarding the exponential growth formula.



(a)



(b)

Figure 5. Exponential growth pattern example by two students in the experimental group

The control group. Of 41 students 7% gave correct examples of exponential growth; one of whom presented an example of a real-life situation (see Figure 6). Sixteen students (39%) were unable to describe exponential growth patterns. Twenty-two students (52%) described exponential growth patterns by writing the growth formula [y= $P(1+r)^n$] and explaining what each variable represents. Two students from this group wrote out the decay formula.

You deposit \$5,000. You get 7% annual interest compounded yearly. what will the accont balance be in 2 years?

5,000 is the principal. 7% becomes a decimal: 0.07 and is divided by 1 and is added to 1 which gives you 1.07. This is the growth factor. Then do 1.07^2 and multiply it by 5,000. you will get 5,724.50. the accont balance will be \$5,724.50.

Figure 6. Exponential growth pattern example by a student in the control group

The way an exponential function was presented to the control group was by introducing the formula and explaining what each variable means. After that, the teacher demonstrated how to write an exponential function by plugging the given information in the equation. This teaching method might be why they answered this question by giving the formula. On the other hand, the experimental group were never given the exponential growth and decay formulas. They were given scenarios where they have to come up with an equation to represent the situation.

Mathematical reflection 3. In this reflection, students were asked to connect the three different representations of an exponential function and explain how to get information from each representation. It consists of three questions:

Question 1. How can you use a table, a graph, and an equation to find the y-intercept and the growth factor of the function?

The experimental group. Of the 44 students 25% gave a detailed explanation on how to find the y-intercept and the growth factor from the table, the graph, and the equation (see Figure 7). While the first student as shown as in Figure 7a managed to get his point through using descriptive words, the second students as shown as in Figure 7b used visual representation to explain his understanding.

Graph - you would find the point that crosses y and that would be y-inter. Then divide 2 points together and that would be growth factor.
Table - you would already have the equation for the table. Plug in X and you get y-inter, divide 2 points and get growth factor.
Equation - ~~y~~ you would plug in X for 0 → zero and find the y-inter. The base of the exponential growth is the growth factor.

(a)

$y = 2^x$	-2	-1	0	1	2
↑ growth factor	0.25	0.5	1	2	4

x2 x2 y-intercept x2 x2

$Y = 5(2)^t$

Y-intercept — Growth rate

In a graph you see when it passes 0 & the growth factor is the increase or decrease.

(b)

Figure 7. How to use the table, equation, and the graph to find the y-intercept and the growth factor by two students in the experimental group.

The remaining 33 students (75%) did not manage to answer the question correctly. They focused on finding the y-intercept, and they suggested making a table of values (see Figure 8a), or substituting zero for x then solving for y (see Figure 8b).

By looking at a table, graph, and equation you can see the growth factor and plug in numbers to find the y intercept.

(a)

You can use X=0 and Solve for Y intercept And use the Equation to find growth factor

(b)

Figure 8. How to use tables, graphs, and equations to find the y-intercept and the growth factor by two students in the control group.

The control group. Of 41 students 22% answered the question correctly. They sufficiently explained how to find the y-intercept and the growth factor from the table, the graph, and the equation (see Figure 9).

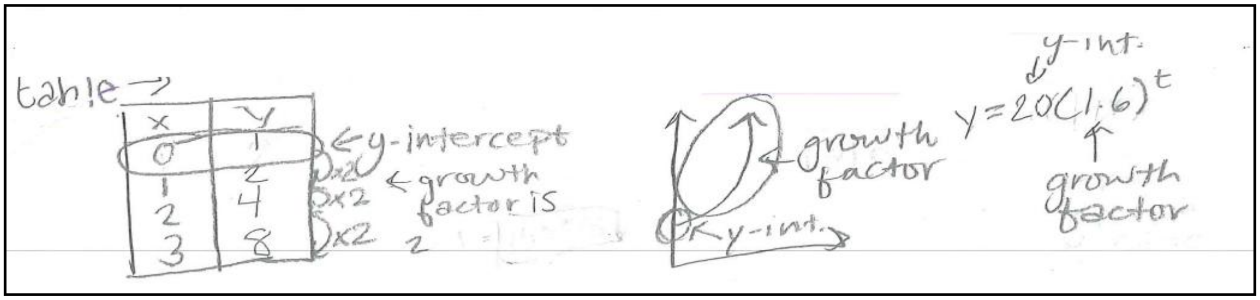
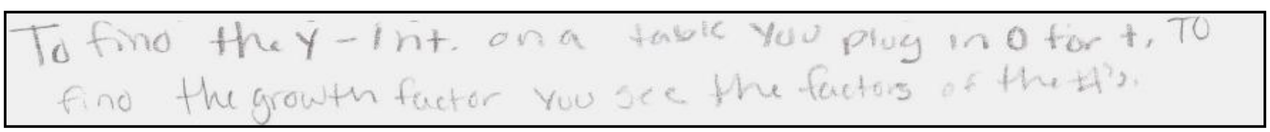
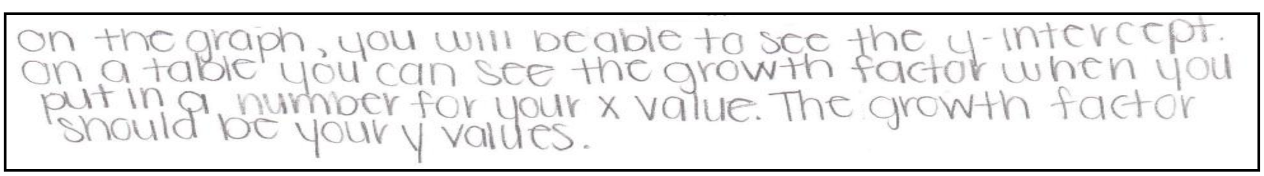


Figure 9. How to use tables, graphs, and equations to find the y-intercept by a student in the control group.

Thirty-two students (78%) did not manage to form an answer. Most of them noted that you simply look at the table, graph, or equation, and plug in the values. Some of them suggested that, to find the y-intercept, you must "plug in 0 or make a table" (see Figure 10a). Others said that you can graph it, and then find the y-intercept (see Figure 10b). They did not specify how to find the growth factor from the graph or the equation.



(a)

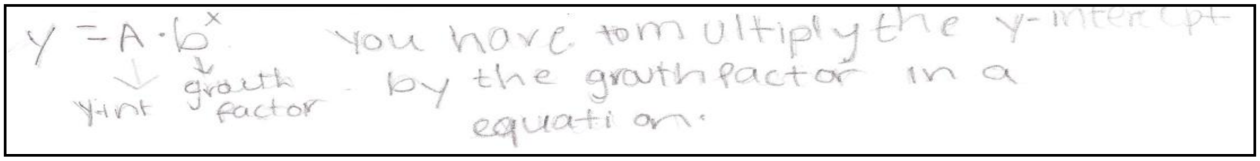


(b)

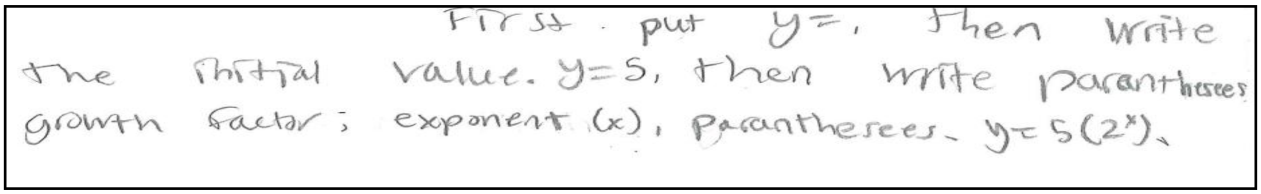
Figure 10. Using graphs, tables, and equations to find the y-intercept and the growth factor by a student from the control group.

Question 2. How can you use the y-intercept and the growth factor to write an equation that represents exponential functions?

The experimental group. Of the 44 students 48% answered this question correctly, some of them used the formula $y=ab^x$ to write an exponential function (see Figure 11a) and some gave a descriptive way to write the equation then gave an example (see Figure 11b).



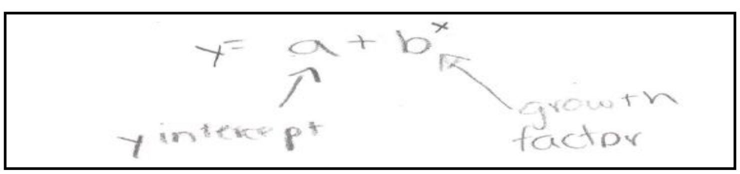
(a)



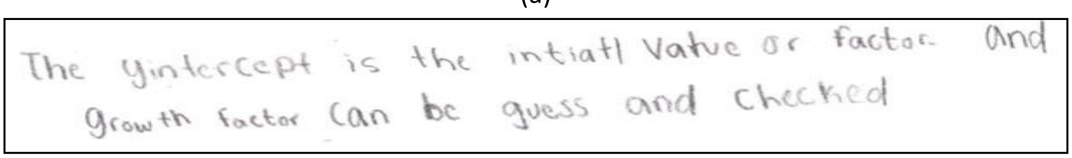
(b)

Figure 11. How to correctly use the y-intercept and the growth factor to write an equation that represents exponential functions by two students in the experimental group

Twenty-one (52%) students did not answer the question correctly. Some of them wrote the formula wrong (see Figure 12a) and some of them provided a description of the equation but never wrote it (see Figure 12b). So far students in this group have been writing equations for different situations for about a week. They are used to writing equations for real-life situations; they have not yet encountered writing equations where the information is given to them in an abstract format with no connection to real life.



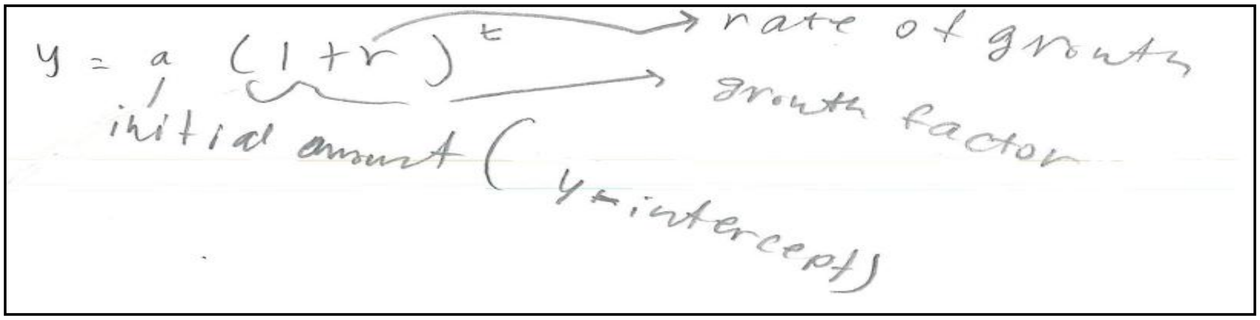
(a)



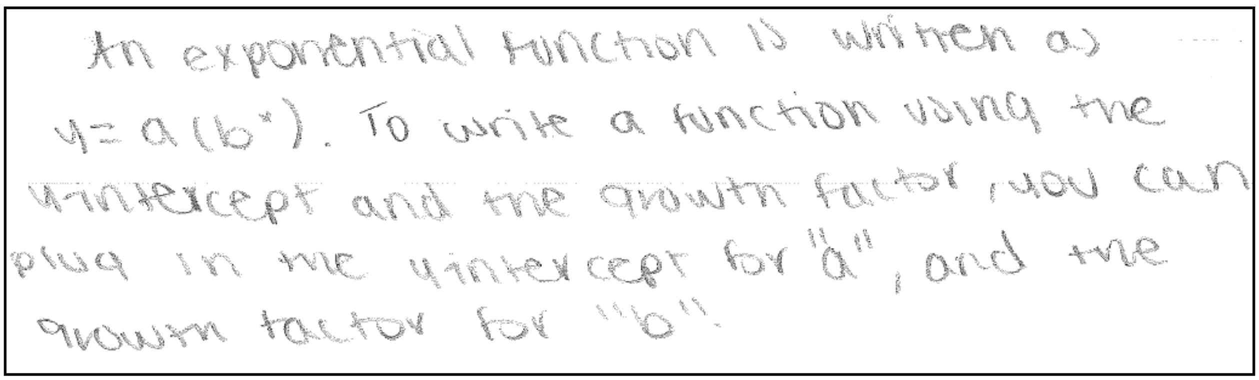
(b)

Figure 12. The mistake in using the y-intercept and the growth factor to write an equation that represents exponential functions by two students in the experimental group

The control group. Of the 41students 39% answered this question correctly. Some of them wrote the formula y= $P(1+r)^t$ and explained what does each variable mean (see Figure 13a). Other students use the form y= ab^x (see Figure 13b).



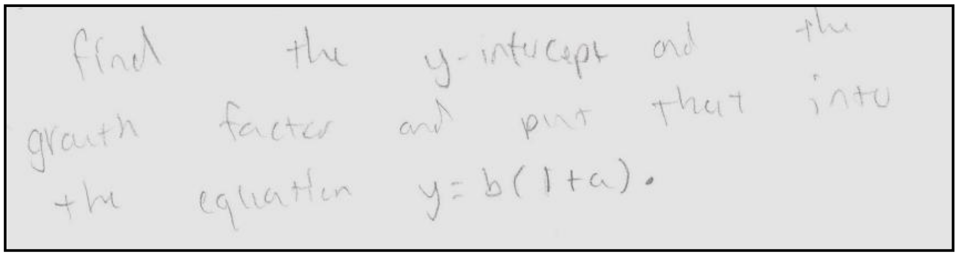
(a)



(b)

Figure 13. How to correctly use the y-intercept and the growth factor to write an equation that represents exponential functions by two students in the control group

Some students referred to the y-intercept as the initial value. This is because this section was focused on interest rate problems. Twenty-five students (71%) did not manage to answer this question correctly, some of them wrote the wrong equation (see Figure 14).

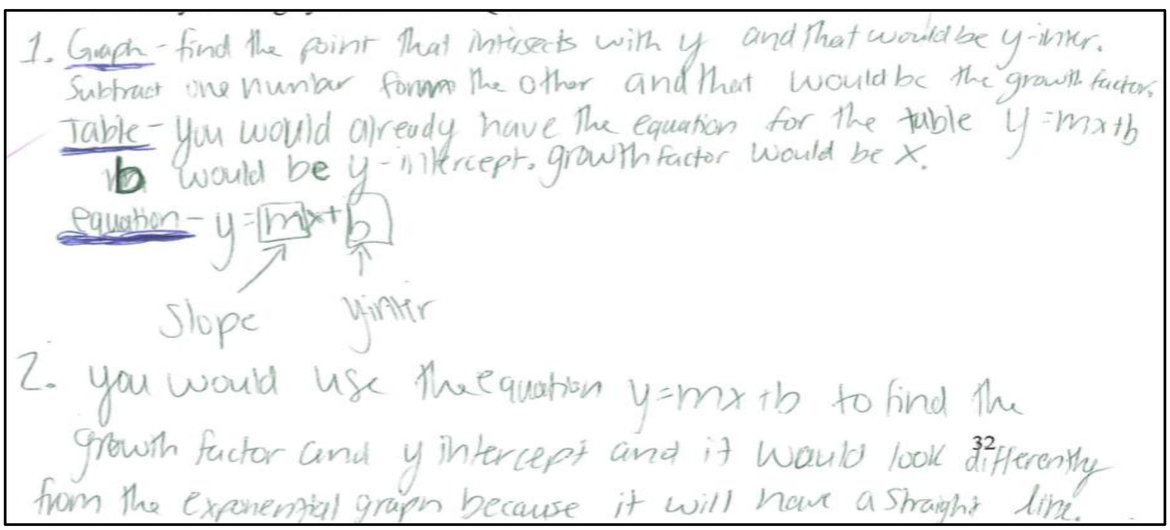


find the y-intercept and the growth factor and put that into the equation y= b(1+a).

Figure 14. The wrong way of using the y-intercept and the growth factor to write an equation that represents exponential functions by a student in the control group.

Question 3. How would you change your answers to questions 1&2 for a linear equation?

The experimental group. Of the 44 students 32% answered this question correctly. They explained that the growth factor is the slope, while the y-intercept is b in the linear equation y= mx+b. They indicated that the growth occurs by adding the slope value, while in exponential functions it occurs by multiplying (see Figure 15). Twenty-eight students (68%) did not give detailed explanations; five of them wrote the slope-intercept form with no explanation, and some of them said that you add instead of multiply.



1. Graph - find the point that intersects with y and that would be y-inter. Subtract one number from the other and that would be the growth factor
Table - You would already have the equation for the table y=mx+b
b would be y-intercept. growth factor would be x.
Equation - y=[m]x+[b]
Slope y-inter
2. you would use the equation y=mx+b to find the growth factor and y intercept and it would look differently from the exponential graph because it will have a straight line.

Figure 15. How to use growth factor and y-intercept to write a linear equation by a student from the experimental group.

The control group. Of the 41 students 34% adequately answered this question. They indicated that the growth factor is the slope, which is added, not multiplied, and the y-intercept is clearly stated in the slope-intercept linear equation formula. Some of them suggested using the slope formula to find the growth factor of linear functions (see Figure 16). The remaining 27 students (66%) did not give a complete answer. Two students suggested plugging the numbers into y=mx+b. Some students said that the x and y increase each by a certain rate, some said that it is a straight line, some suggested graphing it, and some said that there is no difference except that linear functions are straight lines.

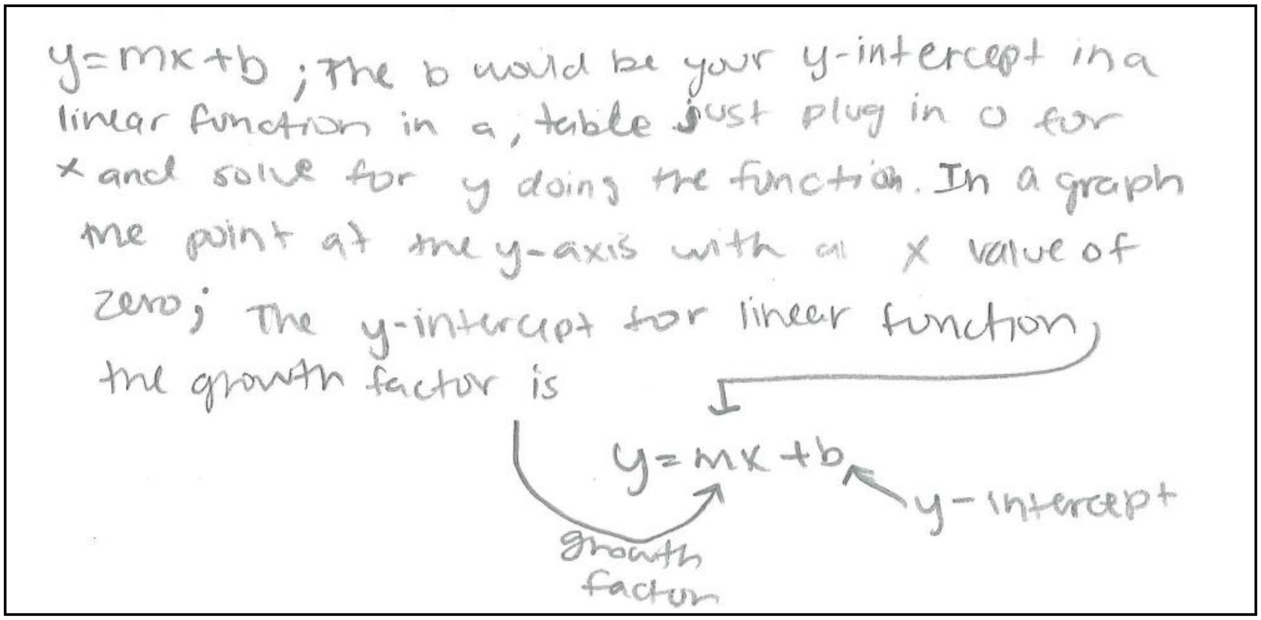


Figure 16. How to use growth factor and y-intercept to write a linear equation by a student from the control group.

Mathematical reflection 4. This reflection asked the students a number of questions to determine if they can differentiate between growth rate and growth factor. It consisted of three questions:

Question 1. Suppose you are given the initial value for a population and the yearly growth rate.

a) How can you determine the population several years from now?

The experimental group. Of the 44 students 34% answered the question adequately, they used the formula $P = ab^x$ and explained that a is the initial value, b is the growth factor that can be obtained by adding one to the growth rate, and t is the time (see Figure 17). Nine students (20%) answered the question partially correct; they suggested using the equation $P = ab^x$, but they made a mistake in calling b the growth rate. Twenty students (46%) suggested writing an equation, plugging it into an equation, or solving, but none of them gave the equation.

By using the Formula AB^n where you plug in the growth rate and initial value. For example if the initial value is 200 and the growth factor is 2.7. So you use this formula as this $200(2.7^n)$.

Figure 17. How to determine the population after a number of years using growth rate and initial value by a student in the experimental group.

The control group. Of the 41 students 44% answered correctly by giving the growth formula and explaining what each variable means. Six students (15%) suggested using the formula to solve the problem, they gave the growth formula without explaining what each variable means (see Figure 18). Seventeen students (41%) did not manage to answer; some of them wrote “idk” which means I don’t know. Some of them said to plug in a number for t but never wrote the equation that we need to plug it in.

$y = a(1+r)^t$ you put the initial amount in place of a, the growth rate in place of r, and the amount of years from now in place of t.

Figure 18. How to determine the population after a number of years using growth rate and initial value by a student in the control group.

b) How is growth rate related to growth factor?

The experimental group. Of the 44 students 16% gave the correct relation between growth rate and growth factor. They indicated that the growth factor equals the growth rate plus one (see Figure 19). The rest of the 44 students (84%) indicated that the growth factor and the growth rate are related, but neither indicated how, or they gave a wrong relation. For example, some said that they are both the same (see Figure 20).

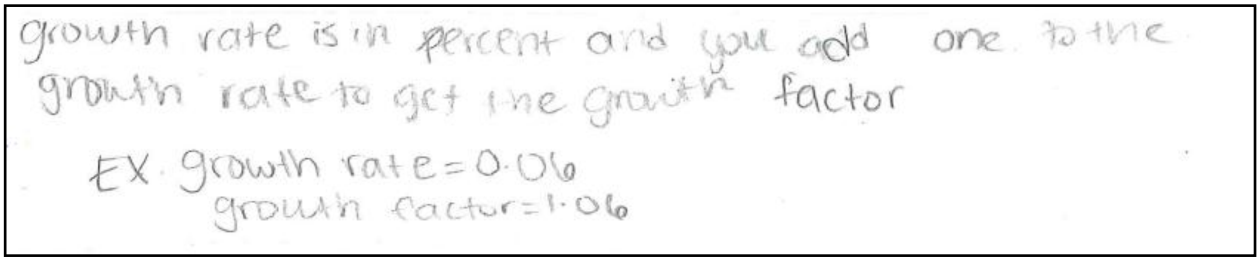

growth rate is in percent and you add one to the growth rate to get the growth factor
EX. growth rate = 0.06
growth factor = 1.06

Figure 19. The relation between growth rate and growth factor by a student in the experimental group.

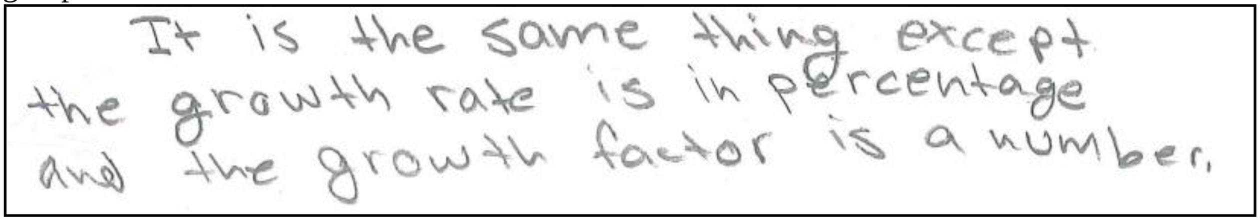

It is the same thing except the growth rate is in percentage and the growth factor is a number.

Figure 20. The wrong relation between growth rate and growth factor by a student in the experimental group.

The control group. Of the 41 students 24% correctly answered this question. They indicated that adding one to the growth rate will give the growth factor or subtracting one from the growth factor gives the growth rate (see Figure 21). The rest of the 41 students (76%) did not answer it correctly. Some of them suggested substituting one into the exponential growth equation, some suggested subtracting the growth rate from one, some suggested changing r from a decimal to a percent, and some suggested that the growth factor is the growth rate added to the original population (see Figure 22).

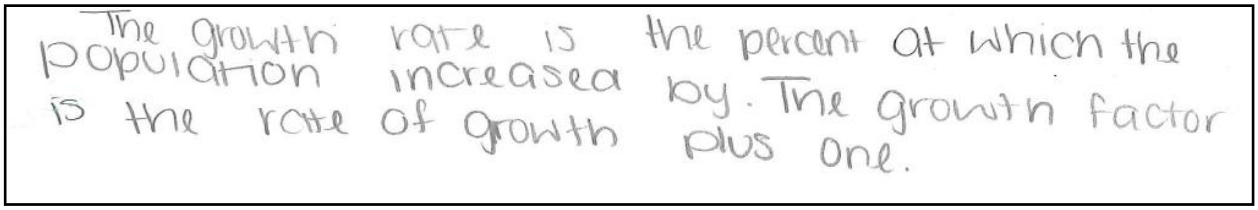

The growth rate is the percent at which the population increased by. The growth factor is the rate of growth plus one.

Figure 21. The relation between growth rate and growth factor by a student in the control group.

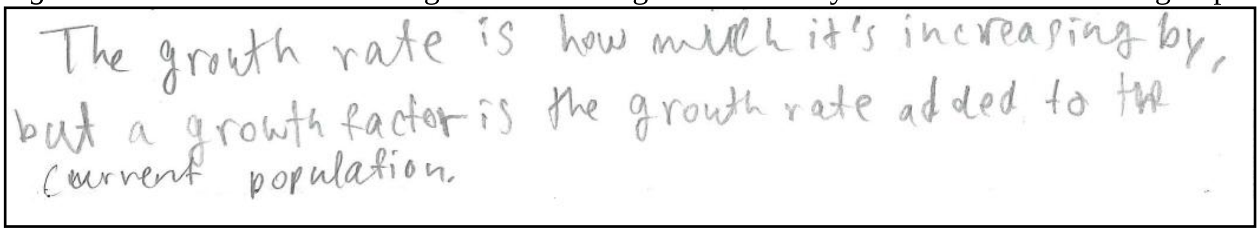

The growth rate is how much it's increasing by, but a growth factor is the growth rate added to the current population.

Figure 22. The wrong relation between growth rate and growth factor by a student in the control group.

c) The third part of this question asked how one can use the previous information to write an equation to model a given situation.

The experimental group. Of the 44 students 41% gave a correct answer. They suggested using the formula P = ab^{t}, and they explained what a, b and t represent (see Figure 23). Eleven students (25%) gave the correct formula, but they said b is the growth rate instead of the growth factor. 15 out of 44 students (34%) did not give any equation.

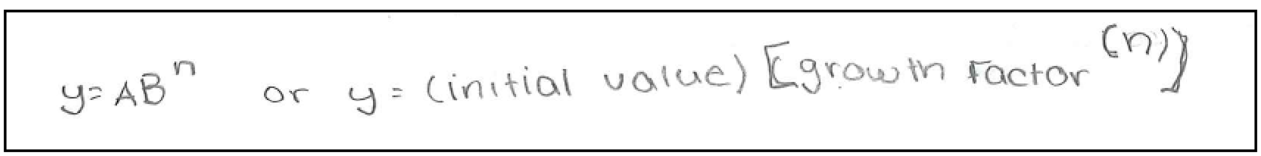

$y = AB^{n}$ or $y = (\text{initial value})[\text{growth factor}^{(n)}]$

Figure 23. An equation to model a given situation by a student from the experimental group.

The control group. Twenty-two out of 41 students (54%) answered this question correctly. They used the formula y= P(1+r)^{t} to model the exponential equation. Eight of them explained what each variable in the equation represents (see Figure 24), while the rest wrote the equation with no explanation. The remaining 19 students (46%) did not explain or write the exponential function model. Some of them wrote the decay formula, and some gave a numerical formula such as 2834(1+0.2)^{10}, and provided no explanation.

a = initial (500)
r = rate of growth (70%)
t = time in years (7)

$y = a(1+r)^t$
$y = 500(1+.7)^7$
$\boxed{y = 500(1.7)^7}$

Figure 24. An equation to model a given situation by a student from the control group.

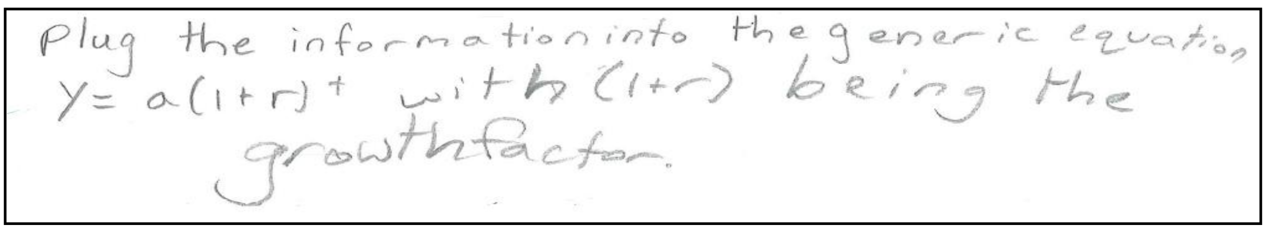
Question 2. Suppose you are given the initial value for a population and the yearly growth factor. The first part of the question was: how can you determine the population several years from now?

The experimental group. Of the 44 students 45% answered this part correctly. Some of them wrote the answer in equation format such as P= (the initial value)(growth factor)^t or $P = ab^t$, and they indicated that a is the initial value and b is the growth factor (see Figure 25). Three students (7%) suggested plugging in the growth factor, the initial value, and the time in the equation, but they never wrote which equation they were referring to. The remaining 21 out students (48%) did not give the correct answer. Some of their suggestions included: writing a linear or exponential equation, making a graph, multiplying time by the growth factor, and plugging the growth factor in the equation. None of these answers were followed by further explanations.

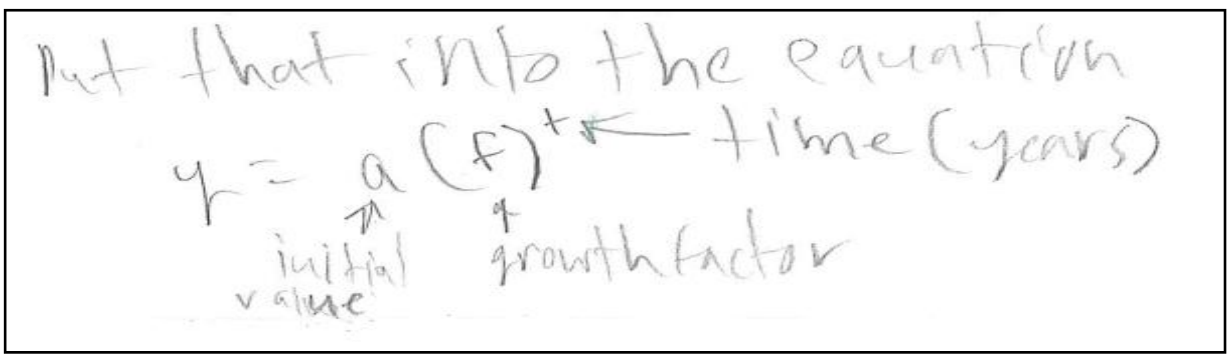
Make an equation based off $y=a(b^x)$. Then plug in the initial value of population (a), the growth factor (b), and the time from now (x), which will get you y. the population.

Figure 25. Using the growth factor to find the population after several years according to one student in the experimental group

The control group. Of the 41 students 39% answered this part correctly. They suggested raising the growth factor to the power of t, with t being years and then multiplying it by the initial value (see Figure 26a). Some of them suggested plugging it in the equation $P = a(f)^t$, and they specified that (t) is time, (a) is the initial value, and (f) is growth factor (see Figure 26b). Two students (5%) partially answered the question correctly. One of them said "the growth factor would go into the parenthesis and it would be simplified." He never indicated what parenthesis he was talking about, and he did not give an equation. The other one suggested using the equation $P = a(1+r)^t$ with no further explanation. The remaining 23 students (56%) did not manage to answer it correctly. They either gave no answer, or they gave wrong answers such as finding the growth rate, or plugging it in the equation: $P = a(1+r)^t$. One student suggested plugging the information in $P = a(1- r)^t$.



(a)



(b)

Figure 26. Using the growth factor to find the population after several years according to two students in the control group

The second part of the question asked students to explain how to determine the yearly growth rate.

The experimental group. Of the 44 students 14% answered this question correctly; subtracting one from the growth factor will give the growth rate (see Figure 27a). Twelve students (27%) listed ways you can find the growth factor, not the growth rate, they suggested dividing the population of a certain year by the population of the previous year (see Figure 27b). The remaining 26 students (59%) did not answer this question correctly. Some suggested using the equation, but did not write it down, and some suggested making a table.

The growth rate is the growth factor minus (-) 1.
Ex. 1.06 − 1 = 0.6
→ growth rate
or 0.06

(a)

If it increases from 100 to 160
the increase is 60 so that would
be 60% which is 1.6 as the growth
rate

(b)

Figure 27. How to determine the growth rate by two students from the experimental group.

The control group. Of the 41 students 24% answered this part correctly (see Figure 28a). They suggested subtracting one from the growth factor. The remaining 31 students (76%) did not answer this part correctly. Some of them suggested plugging one instead of t into the equation $P = a(1+r)^t$, or moving the decimal in the growth factor two places to the right. One student said to add or subtract one from the growth factor (see Figure 28b).

yearly growth factor − 1 = yearly growth rate

(a)

By adding or subtracting 1 from the growth factor.

(b)

Figure *28*. How to determine the growth rate by two students from the control group.

Question 3. Suppose you know the equation that represents the exponential function relating the population P and the number of years n. How can you determine the doubling time for the population?

The experimental group. Of the 44 students 43% answered this question correctly. They suggested guessing the number of years, then plugging it into the equation $P = ab^t$ until you find the year that will give you double the original population (see Figure 29a). The remaining 25 students (57%) did not manage to answer it correctly. Some of them suggested using $P = 2^n$ (see Figure 29b), or multiplying the equation by 2, but did not give the equation they referred to. Some suggested graphing the equation, but no equation was given as well.

You can use guess and check. Or make a chart until it doubles.

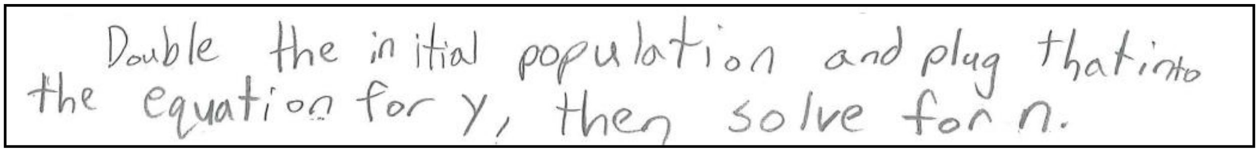
(a)

$P = 2^n$: use that formula because 2 represents doubling it.

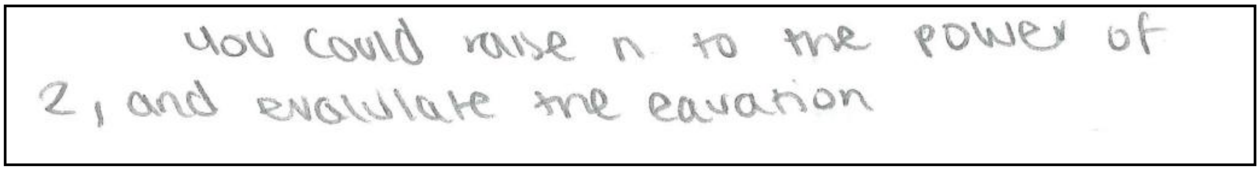
(b)

Figure 29. How to determine the population doubling time according to two students in the experimental group.

The control group. Of the 41 students 12% answered this question correctly. They suggested doubling the initial value, plugging that number into P then solve. They never indicated how to solve it (see Figure 30a). The remaining 36 students (88%) did not manage to answer it correctly. Some of them suggested making the formula from $P = ab^n$ to $P = ab^{2n}$, raising n to the power of 2 (see Figure 30b), evaluating an equation that was not given, multiplying the answer by 2 to double it, and doubling the years.


Double the initial population and plug that into the equation for y, then solve for n.

(a)


you could raise n to the power of 2, and evalulate the eavation

(b)

Figure 30. How to determine the population doubling time according to two students in the control group

Observations. Students in the experimental group were active recipients of the knowledge. They were engaged in the construction of their learning. It enabled them to answer extension questions that were different than what they did in class. They understood the concept, so they managed to expand their knowledge by answering questions with new ideas. On the other hand, students in the control group were not actively involved in their learning. They were passive recipients of the information which limited their ability to expand on their knowledge to solve problems that they've never encountered before. The focus with the control group was on the procedure, so they were better at answering mathematical reflection problems that focused on the procedures. The focus of the experimental group was on conceptual understanding, they were better at answering mathematical reflection questions that focused on conceptual knowledge.

The quantitative data results provided evidence to support my observation, which is that, when students are actively engaged in their learning, they acquire the knowledge in a way that helps them expand it and achieve better (the experimental group). The results of the mathematical reflections also support my observation, which states that: If the focus is on procedural knowledge then students tend to do better answering procedural questions (the control group). If the focus is on conceptual understanding, then students tend to do better answering conceptual questions (the experimental group).

Students in both groups reviewed simplifying exponents, scientific notations, writing exponential growth and decay equations, and graphing exponential growth and decay. The following is detailed analysis of my classroom observations to each group:

The control group. From my classroom observations, I noticed that the control group learned each topic without connecting it to other topics or to real-life. The students were sitting quietly most of class time, either doing individual work or listening to the teacher going over some problems. Some students were engaged and participating while others were just looking around. The traditional teaching method started by giving the students the general rule, solve a couple of examples, then have them solve similar problems as practice. Later students would be expected to apply the rule when appropriate. The students were supposed to take the first entry ticket (see Appendix E) on the third day of the study. That was delayed because of the teacher. "The teacher decided to postpone giving the entry ticket because she did not give them similar problems" (Observation notes, 12/3/2015). Then, students were given a worksheet with problems similar to the problems in the entry ticket. As I was circling around, I noticed several students were having a hard time solving the assigned problems. If the question was not similar to what they practiced in class, some students had a hard time solving it. This was an indication that not all students acquired the anticipated knowledge.

The curriculum started each lesson with what was referred to as an activity. The activity was a regular example, focused on procedural knowledge, in which students were following the directions step-by-step to solve the problem. The teacher directed the students to work independently on the given examples in the book. After the students finished, the teacher discussed the examples with the students and projected her solution to the examples. Students had the rest of class to work on homework.

All the rules of simplifying exponents were put in one lesson in the curriculum. It was not broken down in a way that allowed students to understand one rule at a time as I've seen in other traditional curriculums. The teacher went over all the rules in one day. It took three extra days of practice to help students become more comfortable with simplifying exponents. In one of the worksheets students have some problems that required using multiple rules to simplify exponents. Most of the students did not manage to do these problems correctly. The book exposed the students to simple simplification problems which made it very hard for them to expand their knowledge to simplifying exponents with multiple steps. This was an indication that they were having a hard time simplifying exponents using multiple rules. They were advised by the teacher to study from the book and were given a packet to practice.

During the Scientific notations lesson, I noticed some students were paying attention and working with the teacher while others were looking around. The lesson covered converting between scientific and standard notations and operations with scientific notation. A couple of days later the teacher asked the students a few questions about scientific notation. Most of the students did not know what to do. The teacher spent the rest of that class reviewing operations with scientific notations.

For exponential functions, students were given the general growth and decay formulas. The teacher explained what each variable represents. This was done in two days and one day was given to graph exponential functions. Students' made a table and graphed exponential growth and decay equations. The teacher took some time at the end of each class to go over the problems and how she solved it. “The teacher asked the students what they noticed about the exponential graph that they graphed. Some students raised their hands and participated while the rest just sat quietly” (Observation notes, 12/8/2015). All the word problems they did were a direct application to the formula in which the numbers were clearly identified.

During the lesson students were asked to write an equation to represent a given situation, they all gave the same equation because they all used the same formula. All the problems they encountered were related to interest and investment, and all the numbers were clearly identified to be plugged in the formula. The curriculum never mentioned anything about growth factor but the teacher repeated the relation between growth rate and percent of growth in every class for three days but no problems was solved in which the growth factor was given instead of the growth rate. When graphing exponential growth and decay, I noticed that students tended to use the same scale, which was one. When the initial value was very big, they used 100 or 1000. They did not work on any problems that required them to use a different scale.

Experimental group. The experimental group learned by solving real-life problems. They were expected to use their prior knowledge to simplify exponents and to write and solve problems. They worked on four investigations, each of which consisted of a number of lessons. Each lesson consisted of several problems. Students worked in groups to solve the problems. Each lesson was a building block that helped students constructs their understanding about writing and graphing different forms of exponential functions. Students explored each problem differently. Different groups gave different equations to represent the same problem. When groups were asked about their answer, they explained why their answer was a correct one as well as how they thought of the problem. I also noticed that all group members were working, and they all participated in the group discussion “all groups were working and discussing the problem, some groups wanted to be the first group to finish" (Observation notes 12/7/2015) and "all groups are engaged in great discussions about how to write the equation that represents the situation" (Observation notes12/9/2015)

For the experimental group, the lessons were a group of problem-solving investigations. In each lesson, students were required to simplify exponents, write exponential equations, and graph exponential functions. Students were expected to know simplifying exponents from the previous year. Unfortunately, they barely did anything in that unit, as it was the last unit in the book. The students indicated that they did not have time to do the exponents unit in seventh grade. The teacher took some class time to review simplifying exponents, and gave the students a worksheet for homework.

They investigated exponential growth in relation to bacteria, mold, and investment. They solved problems that required them to do the mathematics and explain the results in terms of the situation. The homework was problem solving situations similar to what was done in class, yet it was an extension. Students had to explain their results in terms of the situation, not just give the numerical answer.

Students worked in groups from day one of the study. At the beginning, it was challenging because they had never learned mathematics in groups. They waited for the teacher to give the information, but when they realized that no information would be given, they worked diligently on the problems. Group members were on task and working at all times. At the beginning of the study, two students did not participate very well. They told me that they wanted the teacher to teach them. Then gradually they participated in their group discussion. By the third day of the study all students were working and on task "students were on task all class time. No student was sitting doing nothing or bored" (Observation notes, 12/3/2015).

Students were allowed to use graphing calculators, so they had to explore and develop a way to use the functions in the graphing calculator to help them solve the problems faster and more efficiently. When students were asked to write equations to represent a situation, different groups came up with different, yet creative, equations. Students started graphing exponential growth and decay from the first day of the study, so they gained experience in determining the scale that best fit the situation. After the students made the table for a doubling pattern, I noticed a number of groups counting the number of squares on the graph paper. Then, they would take, for example, 2^6 and divide it by that many squares to figure out the scale. "When students graphed 4^x, some groups counted the squares on the graph paper to help them determine the scale" (Observation notes, 12/7/2015). Their ability to figure out a suitable scale for each situation was an indication that they understood graphing exponential functions.

Interview responses. One question of the interview focused on assessing students' learning and achievement. Students were asked to choose an activity that helped them derive the most knowledge. Students in the control group chose an activity but were unable to provide a convincing reason as to why it helped them learn the most. Their answers were very simple, such as "I learned best this way". I could not get more out of them. In the experimental group, each student chose an activity that helped him/her learn best and then voluntarily explained in detail why they found it helpful. The effect that the teaching method had on the experimental group was profound, and it left students with very much to say.

They were asked the following question: What activities, lessons, and methods of teaching helped you learn the most? Was it taking notes, solving problems in class, discussing the problem with your group, doing homework, listening to the teacher? Why?

The control group. Eight students were interviewed (20% of 41). Students' responses were divided into two groups; one group consisted of three students out of eight. They thought that taking notes was the most beneficial for them during this unit, saying, "Taking notes, writing helps memorize and learn" and "Homework, studying, and taking notes." The other group indicated that solving problems with the teacher and discussing the problems as a class was the most beneficial to them. During exponential growth and decay, the teacher discussed some problems on the board with the students. Most of the students were engaged and on task. Some students indicated that this activity helped them learn the most. They said, "Solving problems and discussing them in groups during class", "Working-out problems on the board", and "Solving and discussing problems." One student benefited from both taking notes and discussing problems on the board. He answered, "Taking notes and doing example problems as a class."

The experimental group. Nine students were interviewed (20% of 44). Students listed multiple things that they thought were helpful. They expanded on their answers and explained why and how their choice was helpful. All of them (100%)found that group work was the most helpful method of learning. They indicated that working in groups helped them understand the problems better. Group work helped them construct their understanding in a way that made the concept make more sense. In addition, they enjoyed learning the concept and building their understanding with the group work rather than the teacher giving the information to them. Their positive response to group work was shown by their responses. Some of them indicated that group work was helpful because "They were able to help me with the problem and give me another perspective", "My group helped me understand that confusing problems can be solved easily", "Discussing problems in class and in groups made it easier to understand". Some of them enjoyed working with a group as one said: "I enjoyed the method of talking with my group because we can all figure it out together". Working with a group helped some students learn and build their understanding "Working with other people trying to figure out how to do this instead of having someone tell you and have to figure it by yourself", "Working in groups really helped me better understand the work. Being able to discuss it with a group made it easier to understand". Group work helped them construct their understanding in a way that made the concept make more sense. They stated that: "My group helps because if someone else gets what I don't, they can explain it to me", and "Discussing problems in class and in groups made it easier to understand". In addition, they enjoyed learning the concept and building their understanding with groups rather than the teacher giving the information to them. They said: "I enjoyed the method of talking with my group because we can all figure it out together".

Some students mentioned other things that were very helpful such as students and the teacher discussing homework problems together when the teacher is checking homework answers "Class discussion and calling on students because it gets you tied to the lesson." Another student believed that homework helped him the most because it was reinforcing what they did in class as a group, he said, "The homework reinforced what I learned in class and gave real logical problems. I know the real life application of the concept". The focus during group work was solving problems, and at no point students copied notes from the board. One student suggested that the teacher should plan time for the students to write notes from the board, saying that "it would've been more helpful to take more notes, not only solve problems."

Students' attitudes

To analyze students' attitudes, the qualitative and the quantitative data were used. The qualitative data was collected from students' reflections, classroom observations, and students' interviews. The quantitative data was collected from the pre and post-attitude survey. The qualitative data revealed that the experimental group improved their attitude more than the control group did. Students' reflections showed that the experimental group had higher percentages (when tallied) in their positive feelings toward the teaching method, the change in their attitude, the improvement in their abilities, and the number of skills they learned. From the classroom observations, I noticed that the level of engagement and excitement in the experimental group was higher than the control group. The students in the control group spent most of class time either taking notes or doing individual work, some students looked like they were bored and were looking around. The students in the experimental group were working in groups, all students were actively involved and no student was bored and looking around. The interviews revealed that the experimental group was deeply affected by the teaching method, and students were aware of that effect. They voluntarily explained why the unit affected them the way they indicated. This was not the case with the control group, as I had a hard time getting an answer to why they thought the unit affected them in the way they indicated.

The quantitative data revealed that there was a significant difference in the attitude between the two groups in the pre-attitude survey in favor of the control group (see Table 6). At the end of the study, the t-test indicated that there was a significant difference between the two groups in the post-attitude survey in favor of the control group. The difference between the post and the pre-attitude survey gave an indication of the amount of improvement in each group. Though, the t-test revealed that there was no significant difference in the improvement, the mean scores shows the average growth in each group. The experimental group students' attitudes were affected by the teaching method, which resulted in their attitudes improving by a mean of 5.6 points. The control group improved by a mean of 1.29 points.

Table 8

Pre-and post-attitude survey comparison between control group and experimental group using the t-test

	Control group		Experimental group			
	Mean	SD	Mean	SD	t-test	*p*
Pre-Attitude	138.32	16.593	121.33	26.64	3.495	0.001*
Post-Attitude	139.61	20.475	127.02	22.465	2.693	0.009*
The difference between pre and post attitude scores	1.29	11.568	5.6	16.247	-1.387	0.169

* $p < 0.05$.
SD: Standard Deviation. The test has 34 questions. Each question has a score between one and five. The scores of all the questions were added to obtain a total score for each student.

Students' reflections. The experimental group had higher percentages in their positive feelings toward the teaching method (see Table 7), the change in their attitude, the improvement in their abilities, and the number of skills they learned. While the students in the experimental group explained in detail why their attitude changed, almost half the students in the control group indicated that their attitude improved but they did not provide the reasons that attributed to this change, it is difficult to attribute changes in attitude to the teaching method. The experimental group provided details which, in large part, were directly related to the teaching method, making a link between attitude changes and teaching method more likely.

Table 9

Reflection questions and percent of positive answers in the control group and experimental group

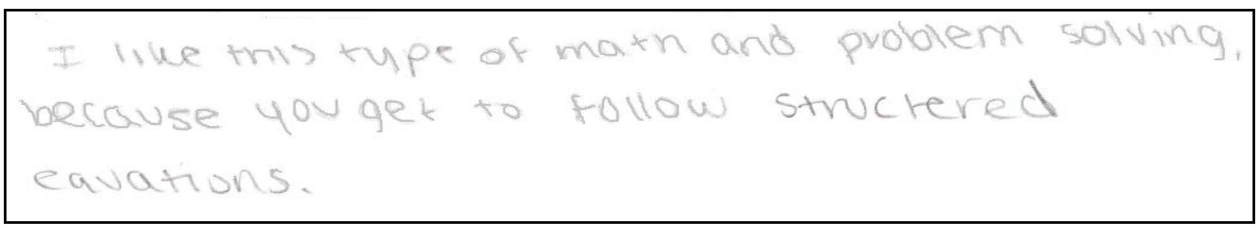
Reflection question	Control group	Experimental group
1. How do you feel about this type of math and problem solving?	24% students said they liked it,	59% students said they liked it
2&3.How would you rate your problem solving ability before and after this unit?	73% students indicated that their problem solving skills improved	91% students indicated that their problem solving abilities improved
4. What skills do you think you learned or improved on by doing this type of math?	They listed a total of seven skills. No explanation on why they think that this unit helped improve each skill	They improved in 13 skills in total. For each skill, they gave a detailed explanation of how did the unit help them improve or did the skill improved.
5. Do you think your attitude toward problem solving or confidence in your ability to problem solving has changed, in what way?	51% of the students indicated that their attitude improved	61% of the students indicated that their attitude improved.

Students answered five questions. The following is a detailed analysis of their responses question by question.

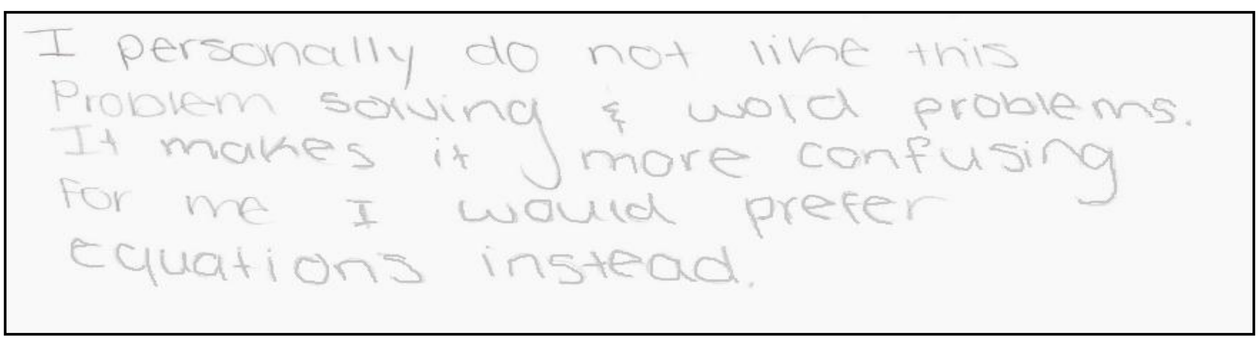
Question 1. How do you feel about this type of math and problem solving?

The meaning of problem solving might be different for each group. Students in the control group were exposed to problems in which they needed to use the formula, and with the numbers clearly identified for each variable. At the end of each homework assignment, they had a question or two involving problem solving. When they did the exponential growth and decay sections, they solved more problems in class and in their homework. Students in the experimental group were exposed to problems from the first day of the study. They were asked to write equations with no formula provided. Students had to examine tables and graphs to come up with equations to represent the given situations. The two groups experienced problem solving in different ways, which leads me to believe that each group interpreted the meaning of problem solving according to their experience.

The control group. Students were very brief in their responses. Ten students (24%) indicated that they liked it (see Figure 31a) some of them felt that it is easy or not hard. Thirteen students (32%) claimed that they did not like it (see Figure 31b), seven students (17%) said its better, five students (12%) did not say anything about how they feel, and six (15%) said that they felt confident (see Figure 32). Some students wrote "all I have to do is memorize the formula", another student wrote "I like this kind of math because you follow structured equations". One said that it was easy, while another said that it was important.



(a)



(b)

Figure 31. The way some students from the control group felt about problem solving.

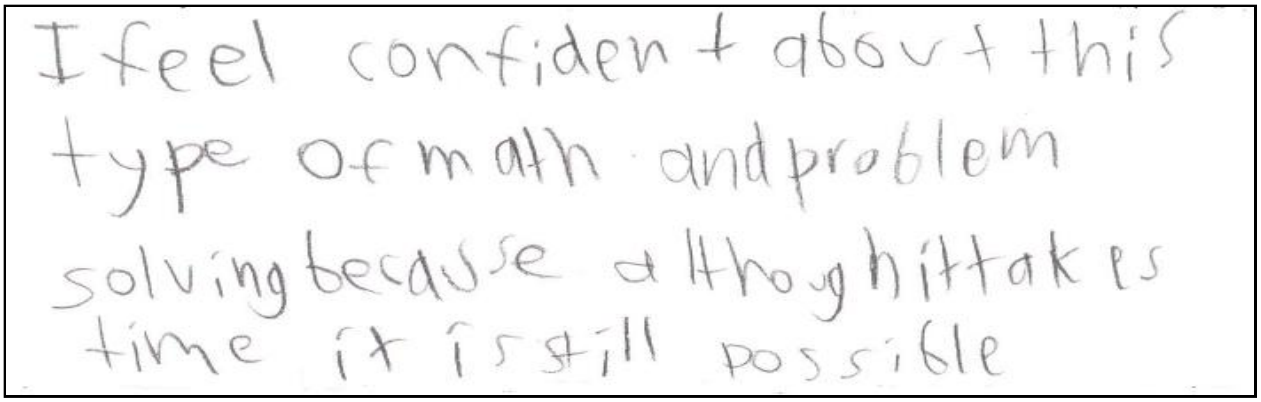


Figure 32. One student from the control group felt confident about problem solving

The experimental group. Students in this group gave lengthy responses to the question. Out of the 44 students 59% said they liked it. Some students indicated that this type of math was fun, that it was easy to understand, and that they liked working in groups. The following are some of their responses: “The problems are easy to understand”, “I feel comfortable; it feels easy to do this type of math”, “You understand what each question is asking”, “I like it because we were working in groups”, “It helps me understand what we are learning”, and “More fun than the other type of math”. The remaining 12 students (27%) indicated that they did not like it. Some of them did not like group work “The learning with the group is not too good”, some thought it was repetitive “It is repetitive and asks similar questions”, and some indicated that they hate math in general. Two students (5%) claimed that they were confident (see Figure 33), and four (9%) expressed no feelings.

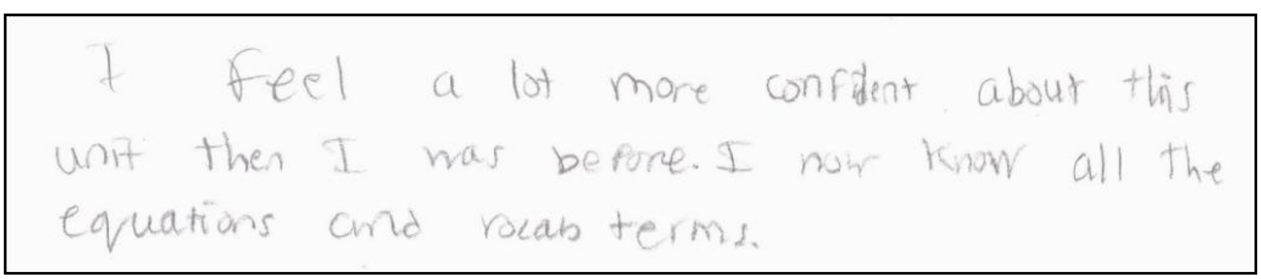


Figure 33. One student from the experimental group indicated that problem solving helped their confident.

Question 2&3. How would you rate your problem solving ability before and after this unit?

The control group. Of the 41 students 73% indicated that their problem solving skills improved. Most of them quantified their problem solving abilities before and after the unit. Their problem solving ability scores range from one to six before the unit and six to ten after the unit. Before the unit, some students indicated that their problem solving ability was good, that it was okay, that they did not know how to solve problems, or that it was not that great. After the unit, they rated their problem solving abilities as better, or they said that they knew how to solve problems now. Four out of 41 (10%) students said it became worse. Seven out of 41 (17%) students said there was no change. Some of them gave a numerical value that stayed the same before and after the unit, and some indicated that they were good before the unit and, after the unit, said that it stayed the same.

The experimental group. Of the 44 students 91% indicated that their problem solving abilities improved. Most of the students quantified their responses. Their problem solving abilities before the unit had a range from one to seven and a mean of 4.75. After the unit, the range was six to ten with a mean of 8.06. Some of them indicated that problem solving was hard for them before the unit, but it became easier after the unit. They indicated that after the unit they understood how to solve them. Others indicated they were good before the unit, and they improved a lot after the unit. Three out of 44 (7%) students said it became worse; two of them gave numerical values that indicated a drop in their abilities. One student said it was good before the unit, but it got worse after the unit, and one out of 44(2%) students said there was no change.

Question 4. What skills do you think you learned or improved on by doing this type of math?

The control group. Students in this group gave vague answers; they did not provide details of how or why they improved certain skills (see Figure 34). They listed the following as the skills they learned; the two formulas, reading and graphing exponential growth and decay, analyzing and interpreting exponential functions, improved at solving word problems, simplifying exponents, and problem solving. Their list was very simple, no more than one sentence per skill. Some students listed only one skill.

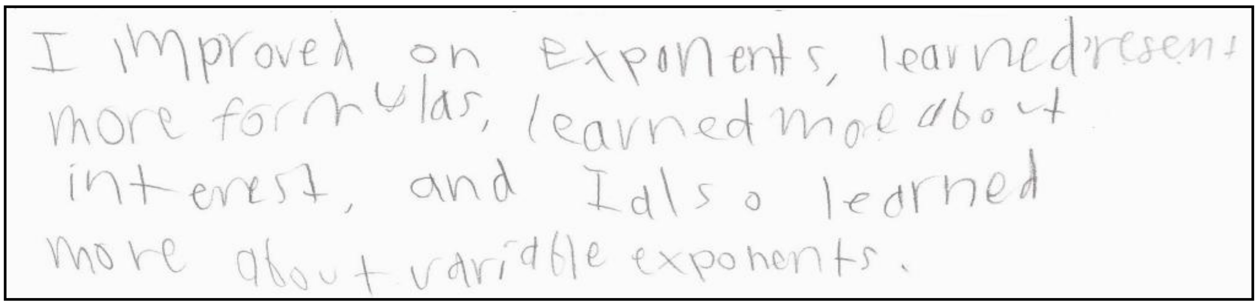

I improved on Exponents, learned presen more formulas, learned more about interest, and I also learned more about variable exponents.

Figure 34. Skills learned by a student in the control group.

The experimental group. They listed the following as the skills they learned; writing and graphing exponential functions (see Figure 35), time management and finding a more efficient way around problem solving, mental math, learning independently and thinking outside the box, creating equations based on story problems and how to solve them, simplifying exponents, improved problem solving abilities, understanding variables and exponents, and rate of decay.

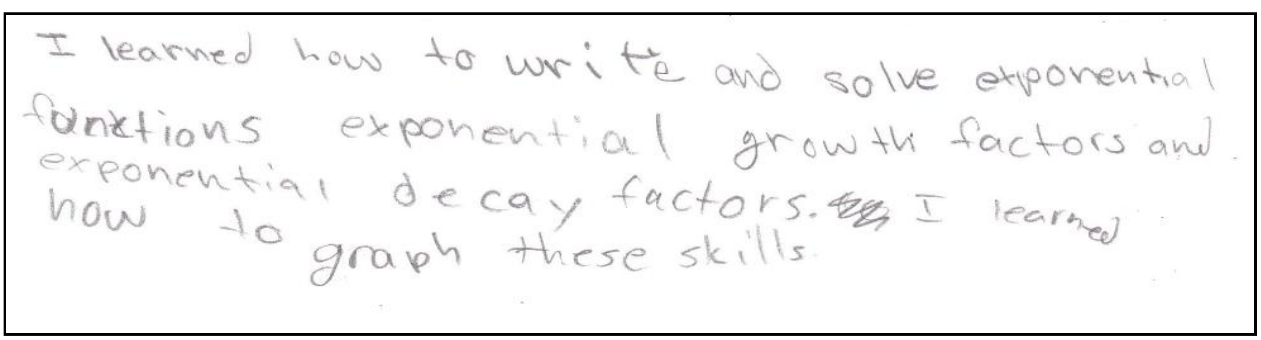

I learned how to write and solve exponential funktions exponential growth factors and exponential decay factors. I learned how to graph these skills.

Figure35. Skills some of the experimental group learned from the unit.

Question 5. Do you think your attitude toward problem solving or confidence in your ability to problem solving has changed, in what way?

The control group. Students indicated briefly what kind of change affected them. No one indicated why their attitude changed; they just indicated that there was a change. Of the 41 students 51% indicated that their attitude improved. The following are some of their comments: “At the beginning of the unit I did not understand this at all; as it progressed I started to have more confidence and get it more”, “I have gotten better at breaking down a problem”, “Now I know the proper methods to solve the problems”, and “I can do more problems at a time and do faster”. Six of the 41 students (15%) indicated that their attitude toward problem solving had gotten worse. They said, “I don’t know what to do because it has gotten harder” and, “I was really good at problem solving but now, I have barely any confidence that I am going to solve it correctly”. Nine of 41students (22%) indicated that their attitude did not change toward problem solving. One student commented “I have always been confident, these problem solving are harder but I can’t still do them and my confident is still the same.” Five out of 41 students (12%) gave no answer.

The experimental group. This group of students stated the change in their attitude and explained the reason for that change. Of the 44 students 61% indicated that their attitude improved. The following are some of their comments: “I like it, it was easier to solve”, “I now understand math more than before”, “Since I solved so many problems correctly it boosted my confidence and helped me solve even more problems correctly”, “It improved my confidence to problem solve because we had to problem solve a lot”, “My attitude toward problem solving is I hate it but my confident in my ability to solve it has gone up a lot, when I first started this unit I was clueless but now I know how to solve the problems", and "I feel now that the equations I write are right. Now I work faster on problems and get them right".

Two out of 44 students (5%) claimed that their attitude decreased; one of them said "My attitude went down. It made me realize that there are more symbols and ways of solving equations that we need to be experts at in life than we think". Six out of 44 (14%) said no change. One of them said, "It did not change because I don't enjoy math and I am still not really good at it", and another student said "My attitude and confidence are the same". Seven out of 44 (16%) did not answer.

Observations for students' attitude. Students' engagement, discussion, and participation are signs that they are part of the learning process and that they are actively involved in constructing their learning. This kind of involvement is the fuel of positive attitudes toward the topic. In my observation, I focused on students' involvement and participation to help evaluate students' attitudes.

The control group. Students in the control group were very quiet throughout each lesson unless the teacher called on them to answer questions. They spent class time taking notes from the board or copying the worked-out examples from the book "class started by the students individually reading the lesson then going over the book examples" (Observation notes, 12/7/2015). Some students were actively doing the work which was an indication that they understood the material well, but others were not actively involved. I noticed that these students were not raising their hands to participate in class discussions, and, as I was circling around, they were struggling to solve the problems. The same group of students were looking around or not paying attention when the teacher explained things on the board or discussed a problem with the class.

The curriculum started each objective with what was referred to as an activity focused on procedural knowledge. The activity was a regular example in which students were following the directions step by step to solve the problem. Students worked individually almost all the time, either on the worked out examples from the book or on solving some problems assigned by the teacher, “the lesson activity was a given equation and a given set of numbers in a table, using the graphing calculator students were supposed to plug the numbers in the equation and record the results. Students were directed to do the activity individually” (Observation notes, 12/7/2015). Some students were on task; others were looking around. As I was circling around, I noticed that the students who were looking around did not finish the given task. At the end of class, the teacher projected her work on the board and discussed the solutions. Only the students who raised their hands were called on to answer. A group of students did not participate and were not following along “a couple of problems were posted on the board; one student went up to solve a problem. A bout half of the students were engaged yet the rest were not even paying attention” (Observation notes 12/4/2015). During class, students betrayed their boredom by constantly looking around. Students were given the exponential growth and decay formulas, and all the problems they worked on involved applying the formula. To some students this was enough, and they understood how and when to apply the formula. Other students had a hard time solving any problem if the numbers were not clearly identified to fit the formula. Students were asked to graph exponential functions by making a table of values. The teacher discussed the y-intercept and the horizontal asymptotes “the teacher projected her work of the problems on the board and asked students what they noticed about the graphs. Some students raised their hands and participated while the rest just sat” (Observation notes 12/8/2017).

It was clear that the teaching method was not the reason why some students were involved in class because no improvement of students' engagement and participation was observed from the beginning of the study until the end. Students who participated and were involved were the same students from the beginning of the study until the end.

The experimental group. The first day was challenging because students were not used to the new teaching method. After the first day, students were on task and working throughout the class period. Four students in total rejected the idea of working in groups at the beginning of the unit, but by the third day they were working and discussing with their groups. From my observations as I was circling around, students were engaged in discussing the problems and their solutions with their group members "One student who was always off task, according to his teacher, was engaged and finished faster than the rest of his group. He said that he liked this kind of math" (Observation notes, 12/3/2015). When the group members were not reaching an agreement on the solution of a problem, some group members would seek help from a group next to them. As the days passed, students got better and faster at solving any given problem. At the beginning, some group members were not very active, but after a few days, all group members became more active in discussing the problems. By the third day, all students in the class were working; no student was doing nothing or looking around "all students were engaged and on task" and "students were on task all class time, no student was sitting doing nothing or bored" (observation notes 12/3/2015). At the end of each class, the teacher would discuss solutions and conclusions with the class. All students were paying attention and eager to participate and discuss their findings "most of the students were raising their hands to answer" (Observation notes 12/9/2015). The teacher would call on any student at any time during class for feedback, and the student would have something to say other than “I don’t know.” Students said “I like this kind of math” multiple times during the study.

Coupled with the greater percentage of students in the experimental group stating that their attitude improved, the observations I made in the classrooms leads me to believe that teaching using authentic problem solving positively affected students’ attitudes.

Interview responses. From the interview answers, I noticed variance in the groups’ answers appears to be related to teaching method. When prompted to explain how mathematics instruction affected them, the control group listed a number of things, yet I could not get them to explain why the unit affected them in the way they indicated. The experimental group had a lot to say. Each student listed several points on how the unit affected them, and then voluntarily explained why the unit affected them in that way. The excitement and engagement was very clear among the experimental group, while the traditional teaching method did not seem to significantly affect the control group. The students were not engaged, and most of them did not appear to be enjoying the unit.

One question in the interview was focused on the effect of the teaching method on students’ attitude. If a student did not give me a detailed answer I followed through clarifying questions.

The question was: Overall, how has your experience in this unit affected you?

The control group. Eight students were interviewed (20% of 41). I grouped their comments into two groups; the positive and the negative. The answers of the positive group were divided into two groups; the first group stated that mathematics is meaningful and useful in real life. They said, “This unit affected me for the better because I believe all math is meaningful and important especially this unit” and “It has shown me that math related to more jobs than I expected". They never explained how they concluded that mathematics is useful in real life after learning the unit. The problems in the unit were geared toward investment and banking; that might be where they saw a good relation between mathematics and real life. The second group indicated that the unit helped them understand exponential functions and increased their confidence in solving them. They said, "This unit has made me more comfortable when solving exponential problems and graphing them", "This unit had a huge effect because it was fun and I learned many new things". The exponential growth and decay formulas were introduced in this unit. The teacher explained how to use them to solve problems. Most students had no problem using the formulas to solve problems as long as they were similar to the problems in the book and the numbers could be clearly identified. The third group, which consisted of one student, did not like mathematics anyway so the attitude remained negative. The student said, "My attitude toward math is negative; this unit did not change my attitude."

Experimental group. Nine students were interviewed (20% of 44). The experimental group, students elaborated on how this experience affected them by explaining why they felt the way they did. Each student said three to four points. I grouped their comments into two groups; the negative and the positive. The negative group consisted of one student who indicated that the experience had a negative effect on him. He indicated that, instead of working for multiple days to come up with a formula, the teacher could have given them the formula in a couple of minutes.

The rest of the students indicated that this unit had a positive effect on them. They felt confident, enjoyed learning this way, and learned a lot. Each of these students listed a number of ways that they were affected by this teaching method. I did not mention confidence in the question, yet eight out of nine students (89%) indicated that this teaching method helped improve their confidence level and attitude toward math “My confidence in solving problems is much higher, now I can do problems with confidence and without any doubts”, and “This math made me more confident since I understood it better. I felt confident in my answers even if I got them wrong. I would like to do this math more often because it makes me feel smart. This way of teaching better suits my learning style”. Three students said that it made them confident in their problem solving abilities “It improved my confident because I feel more comfortable with my answers because I worked with a group.”

Students talked about how they felt toward mathematics class. They indicated that, during this teaching method, they enjoyed mathematics class and they were looking forward to it. They also said they saw that mathematics was useful and related to real life “Math is used in careers and in real life; this unit gave me a different perception about math”. They solved different type of problems such as investment, bacterial growth, and carbon dating. Students worked as groups to construct their learning. They were engaged in solving the problems and discussing it with their group “This type of math makes me look forward to math class”, “This experience made me work harder because it is fun and interesting”, and “Learning this way has really engaged me so much in class than usual. I am actually learning something.”

The students in the experimental group were active recipients of the knowledge. They constructed their understanding which made it easy for them to reflect on their learning and determine which activity was most helpful and why. The students in the control group learned by receiving the information from the teacher, the lessons were not designed in a way that require students to be involved in building their understanding. This made it hard for them to reflect on their learning to determine which activity helped them construct their understanding.

CHAPTER 5 CONCLUSION

This chapter is divided into six sections. I begin by reviewing the results for the two questions of this research. A summarized explanation of the finding for each question followed by a detailed explanation is provided. The results are divided into two parts; one to answer the first question which is about achievement, and the other to answer the second question which is about attitude. In the third section, I provided a relationship between the literature review and the findings of this study. This section is also divided into two sections; one sub-section is focused on the relationship between the literature review and achievement, while the other sub-section is focused on the relationship between the literature review and attitude. The fourth section is focused on the limitations of this study. The fifth section discusses how the results of this study can impact teachers and researchers in the future. Questions and future recommendations is discussed in the last section.

Summary of Results

The purpose of the study was to answer the following two questions:

1. How is students' achievement affected by the use of authentic, problem-based learning when teaching exponential functions?
2. How is students' attitude affected by the use of authentic, problem-based learning when teaching exponential functions?

I will first give a summary of my findings then provide a detailed explanation of my findings for each question.

To answer the first question, the results of the pre and post-tests, quizzes and entry tickets, mathematical reflections, observations, and interviews were used. The results of the qualitative and the quantitative data revealed that the students in the experimental group were positively affected by the teaching method more than the students in the control group. Using authentic problem solving helped students achieve more than the traditional teaching method did. Before the unit, the two groups had no significant difference in the pre-test scores. After the unit, the post-test data revealed that there was a significant difference in favor of the experimental group. The experimental group scored higher in their quizzes and entry tickets.

To answer the second question, the pre-and post-attitude survey, classroom observations, reflections, and interviews were used to provide information about students' attitudes. The quantitative data revealed that the control group was significantly higher in their attitude before and after the study, but the mean improvement in the experimental group was higher than the control group. The qualitative data revealed that students in the experimental group improved their attitude more than in the control group.

Interpretation of Data

Achievement

To measure and compare the change in student achievement, qualitative and quantitative data was collected. The quantitative data was collected from the pre-and post-tests, quizzes, and entry tickets. The pre-and post-tests consisted of nine questions, each of which contains multiple parts. The t-test of the pre-test revealed that there was no significant difference between the two groups in five questions, and that there were two questions significantly different in favor of the control group. Two questions had no test results because everyone in both groups answered it wrong. We can assume that the two groups were almost at the same achievement level before the unit. The t-test of the post-test revealed that the there was a significant difference in favor of the experimental group in six questions and no difference between the groups in three questions. To measure the improvement in each group, the difference between the pre-test and the post-test scores were calculated. The t-test revealed that there was a significant difference between the two groups in favor of the experimental group. It can be concluded that using authentic problem solving to teach exponential functions helped students improve their achievement more than the traditional teaching method did.

Entry tickets were given to students after they finished learning each objective in the unit at the beginning of class on the next day. Their purpose was to evaluate students' knowledge of the lesson. For each group, the questions in the quizzes and entry tickets were similar to the questions in the homework they worked on. To help evaluate student achievement in both groups, the mean and median were calculated for each quiz and entry ticket. The experimental group had 92% of the scores greater than 70%, while 40% of the control group scores were greater than 70%. The experimental group had seven assessments in which the median was 100%, while the control group had two assessments with a median of 100%. This shows that at least half of the students in the experimental group scored a perfect score in seven of the assessments. The control group had half of the students score a perfect score in only two assessments.

The qualitative data was collected from mathematical reflections, observations, and interviews. To help achieve a better understanding of their thinking process, students were given problems in which they had to explain why they solved a problem in a certain way. Students in both groups answered four mathematical reflections, each of which consisted of a number of problems. The percentage of students in each group who answered correctly was calculated. The experimental group had a higher percentage in seven questions, while the control group had a higher percentage in six questions. When the questions were focused on procedures, the control group answered them correctly. When students were asked to explain their reasoning and their understanding, to compare and contrast, or to demonstrate their understanding of the concept, the experimental group did better. Using problem solving approach helped students construct their learning of the concept in a way that helped them answer in-depth questions. On the other hand, using the traditional method helped students have a strong procedural understanding.

In the experimental group, I observed that students were actively involved in their learning. They were engaged with their groups discussing problems. This helped them understand the concept in-depth which thus enabled them achieve higher than the control group, who were mainly passive in the classes. Students in the control group spent their class time either taking notes or doing individual assignments. This way of learning did not help students understand the concept in desired depth, which limited their ability to expand on it in order to answer questions differently than what they had experienced. The focus in the control group was on procedural knowledge, so they did better answering procedural questions, while the experimental group focused on conceptual understanding, so they did better in answering conceptual questions.

During the interview, students were asked to choose an activity that helped them learn the most. Students in the control group chose an activity, but they did not explain why it helped them learn the most. On the other hand, students in the experimental group chose an activity and voluntarily explained why it helped them learn the most. Student involvement in their learning helped them reflect on their learning and thinking process. Since this was only visible in the experimental group, it can be assumed that the teaching method was the reason for student involvement and engagement, which resulted in more reflection on their own learning and Metacognitive activity.

Attitude

To measure and compare the change in attitude, quantitative and qualitative data were collected. The quantitative data was collected from the pre-and the post-attitude survey. The two sample t-test for the pre-attitude survey as well as for the post-attitude survey revealed that there was a significant difference in favor of the control group. Attitude growth was calculated from the difference between the pre-attitude survey and the post-attitude survey. The t-test indicated that there was no significant difference between the two groups. However, the average growth of the students in the control group was 1.59 points, and the average growth for the experimental group was 5.6 points. This was an indication that the growth was not enough to be measured by the t-test, but the students in the experimental group improved in their attitude more than the students in the control group.

The qualitative data was collected from students' reflections, classroom observations, and students' interviews. Students in the experimental group showed more improvement in their positive feelings toward the teaching method, the change in their attitude, the improvement in their abilities, and the number of skills they learned.

In the reflections worksheets, students had to answer five questions regarding their feelings about the teaching method as well as rate their problem solving abilities before and after the unit, list the skills they improved in, and explain whether their attitude or problem solving abilities have changed after the unit. Students in the control group were very brief in their answers. Some of them wrote "all I have to do is memorize the formula" as an answer to how do you feel about the teaching method. They gave a numerical value to rate their problem solving abilities before and after the unit. Out of 44 students, 73% indicated that they improved. They provided no explanation to their improvement. To answer the last question about the skills, they provided a very simple list, with no more than one sentence per skill. Some students listed only one skill, which were mostly focused on procedural knowledge such as the two exponential formulas, solving word problems, and simplifying exponents.

Students in the experimental group gave more explanation to their answers. The focus in their answers for the first question was group work. More than half of them indicated that they liked group work, and they explained why they liked it. The rest of the students attached an explanation to their response, which helped explain the change in their attitude. To answer the questions regarding rating their problem solving abilities, 91% of them indicated that they improved. They provided detailed explanations about their improvement. To answer the question regarding what skills they learned, each student listed multiple skills. The skills were focused on conceptual understanding such as more efficient ways around problem solving, mental math, learning independently and thinking outside the box, and creating equations based on story problems and solving them. The effect of the teaching method left students with a lot to say about what they learned, their problem solving abilities, and the skills they acquired.

Students' involvement was the focus of classroom observations. The control group was not actively engaged; they were either taking notes or individually working on assigned problems. The experimental groups were working in groups actively engaged by solving problems and discussing the solution with their group members. Using authentic problem solving to teach exponential functions helped students be more actively involved, which helped improve their attitude. This was clear from the observations in addition to what they stated in their reflections. On the other hand, the observations led me to believe that the traditional method did not clearly increase students' involvement in class. The students who participated in class stayed the same from the beginning of the study until the end.

Students' interviews were conducted with 20% of the students from each group. They were asked the following question: "Overall, how has your experience in this unit affected you?" When prompted to explain the reasoning behind their answers, the control group did not provide a detailed explanation. On the other hand, the experimental group voluntarily explained why and how they were affected by the unit. The teaching method affected them and changed the way they learned mathematics, which left them with a lot to say. Using authentic problem solving helped the students focus on their learning, which positively affected their attitude.

Relationships between Results and Research Literature

Students Achievement

We see the results of studies comparing procedural versus conceptual focus in instruction reveal that these different foci support student learning differently. Deslauriers conducted a study in 2008 to compare the effect of teaching in two different ways: conceptually-focused instruction (CFI) and procedurally-focused instruction (PFI). Deslauriers indicated that "both types of instruction were beneficial for students. Students in CFI group developed conceptual understanding with brittle skill acquisition and students in PFI group developed problem-solving skills with fragile understanding of concepts" (p. 199). Fernandez (1994) concluded that:

> the children who watches the skills-building lesson were better able to carry out the procedures being taught, their acquisition of these procedures does not seem to have been grounded in having processed the lesson by trying to understand it. The children who watched the sense-building lesson on the other hand seem to have been more actively trying to understand the instructions by attempting to form a coherent representation of it (p. 68).

This dissertation study's results are in line with the results of Deslauriers (2008) and Fernandez (1994). Procedural knowledge was the focus of the traditional curriculum that was used with the control group, while conceptual knowledge was the focus of the curriculum that was used with the experimental group. The results of the mathematical reflections revealed that if the question required students to demonstrate their procedural knowledge, the control group had a higher percentage of correct answers. If the question required students to demonstrate conceptual understanding, to explain their reasoning, or to compare and contrast ideas, the experimental group had a higher percentage of correct answers.

Although both conceptual and procedural knowledge are beneficial for students, one of these foci improves student achievement more than the other. From the literature review, the use of problem-based curriculum seems to take the lead in increasing student achievement. Pesek and Kirshner (2000) indicated that students who learned to focus on procedures will focus on manipulating variables. The main goal for these students becomes knowing how to solve a problem not why. In other words they do not focus on conceptual understanding but all their focus is on the procedure. The problem with this kind of understanding is that they did not reason or make sense of what they were doing which makes their knowledge shallow and easy to forget.

The findings of this dissertation support the findings of Pesek and Kirshner (2000). Teaching with a focus on conceptual knowledge help students improve their achievement more than teaching with a focus on procedural understanding. In this study, students in the experimental group were taught with the focus on conceptual understanding, while students in the control group were taught with the focus on procedural knowledge. The achievement for the experimental group was significantly higher than that of the control group.

Using problem solving approach to teach help students construct their understanding of the concept in unique way that fits their prior knowledge. Multiple studies concluded that this way of learning helped students improve their achievement more than using a curriculum that focuses on procedural knowledge (Bull, 1993; Elshafei, 1998; Pinzker, 2001). Bull (1993) indicated that "students that were taught with an emphasis on the four-step method of problem-solving, a method that addresses the personal strengths of students, improved significantly more in mathematics than students taught through more traditional methods"(p. iv).Elshafei (1998)concluded that "students solving problems in groups preformed better and generated more plausible solutions than traditionally taught students, which is particularly significant for higher-level academic achievement" (p. vi). Pinzker (2001) studied the effect of replacing the traditional method of teaching by using cooperate learning, alternative assessments, and journal writing on students achievement and understanding, she found that " the number of achieving students earning an A,B, or C increased from 19 students to 26 students, a 36% increase" (p. 27).

The finding of this dissertation supports the findings of Bull (1993), Elshafei (1998), and Pinzker (2001). In this study, students in the experimental group were taught using a problem solving-based curriculum, while the control group used the traditional method. The qualitative data revealed that students in the experimental group were able to explain in details the reasoning behind each step of their solution, while the control group used vague statements to explain their reasoning. The quantitative data of this study revealed that there was no significant difference in students' scores in the pre-test. After the unit, the t-test revealed that there was a significant difference in student achievement in favor of the experimental group.

Multiple studies focused on the effect of using authentic problem solving on students' understanding (Choi, 1995; Kirschner, Sweller, & Clark, 2006; Olson, 1998). The use of authentic problem solving helps students relate the math they learned to real life. From the review of the literature, the use of authentic problem solving help students construct a deep understanding of the concepts. Choi (1995) advises, “real-world problems are interwoven in everyday contexts. Therefore, when learning tasks are situated in everyday contexts which provide meaning and relevant experiences, the transfer of knowledge should increase” (p. 8). Kirschner, Sweller, and Clark (2006) claimed that a main property of the problem solving approach is that “they challenge students to solve “authentic” problems or acquire complex knowledge in information-rich settings based on the assumption that having learners construct their own solutions leads to the most effective learning experience” (p. 76).

The results of this dissertation support the findings of Choi (1995), Kirschner, Sweller, and Clark (2006) and Olson (1998). The experimental group in this study used authentic problem solving to help students understand exponential functions. From the results of the mathematical reflections, the qualitative data showed that the experimental group scored higher than the control group in questions that required in-depth knowledge of the concept. The experimental group solved authentic problems where they were given no formula to substitute the numbers in. They have to make sense of the problem and try multiple ways to solve it. On the other hand, the control group solved some problems about interest rates and investment in which the numbers were clearly defined to be plugged in the equation. They did not have to try multiple methods to solve the problem. The effect of authentic problem solving on student’s achievement was also the focus of a study conducted by Olson (1998). Olson indicated that “It appears that through clearly defined expectations and thorough analysis of various student responses, performance on realistic application problems have significantly improved” (p. 26).In this study, the quantitative data revealed that students who used authentic problem solving achieved more than students who learned using the traditional method. Students in the experimental group used authentic problems to help construct their understanding of exponential functions. They explored investments, bacterial growth, the decay of medicine in the blood stream, and the half-life of carbon. On the other hand, the control group focused more on applying the formulas. They only explored growth and decay in investments. This limited their ability to relate what they learned to their lives.

Group work was not part of this dissertation's focus, but it was part of students' daily work in the experimental group. According to the qualitative data, students in the experimental group expressed the positive effect of group work on their attitude and achievement. From the literature review some studies looked into the effect of group work on students' understanding and achievement (Adamson, 2005; Boaler, 2008). The use of group-work and cooperate learning help students construct their knowledge and understand the concept which results in improving their achievement. Adamson in 2005 indicated that "a classroom environment that supports classroom discourse can aid students in their sense-making efforts" (p. 207). In 2008, Boaler conducted a study to determine the effect of problem solving and group work on students' achievement. She indicated that this method of teaching forces the students to use what they know to construct new procedure, and they will construct their conceptual understanding as well as their procedural fluency in a way that makes sense to them. This kind of construction enables them to expand their knowledge. In this dissertation, students in the experimental group were engaged in group discussions to determine the best way to solve the problem. They were able to solve problem that required them to expand their knowledge. On the other hand, students in the control group were listening to the teacher, taking notes, or doing individual work, which did not give them the tools to help them expand their knowledge in order to solve problems they had never encountered before.

Some studies indicated that improving conceptual understanding will improve procedural knowledge. Johnson and Alibali (1999) indicated that "children who received conceptual instruction were just as likely to learn a correct procedure as children who received procedural instruction" (p. 186). The result of this study does not agree with Johnson and Alibali because the experimental group did not manage to be at the same level as the control group in their procedural knowledge. The control group was more proficient in solving problems that focused on procedures, while the experimental group was more proficient in solving conceptually focused problems.

Students' Attitude

The effect of using problem solving and group work on students' attitude was the focus of a study conducted by Wade (1994) and a study conducted by Curtis (2006). They both concluded that using problem solving and cooperative learning positively affected students' attitude. Wade indicated that "qualitative data revealed that the constructivist-based mathematics problem solving instructional program caused a positive shift in students' attitude toward mathematics problem solving." (p. 119).Curtis indicated that:

> Although the attitude inventory did not show a large change in students' attitudes, the qualitative data points toward the results that most students found at least one of the teaching strategies impacted their attitude of being a student of mathematics with regards to confidence, motivation, value, and enjoyment (p. 155).

The results of this dissertation agree with the findings of Wade (1994) and Curtis (2006). The t-test of the survey data revealed that there was a significant difference in favor of the control group in both the pre- and the post-attitude surveys. There was no significant difference in the attitude improvement between the two groups. However, the average growth of the student' attitude in the control group was 1.59 points, and the average growth for the experimental group was 5.6 points.

Most students view mathematics as a topic that has no relation to their lives. A study conducted by DeBay (2013) as well as a study by Adamson (2004) questioned whether teaching using realistic problems help shift students' negative views toward mathematics.

DeBay indicated that "by having students interest increase through solving mathematical tasks that are rooted in meaningful, real-world contexts; students' belief that they can succeed in real-world mathematical tasks; and a shift in students' beliefs regarding the definition of 'doing mathematics"(p. v). Adamson stated that "I have long believed that connecting mathematics to real-world settings would help students overcome their negative feelings toward mathematics and serve to help students learn" (p. 73).

The results of this dissertation agree with the findings of DeBay (2013) and Adamson (2004). The qualitative data revealed that students in the experimental group improved their attitude more and became more positive. Only 73% of the control group indicated that they improved their attitude compared to 91% of the experimental group. Having students solve meaningful problems that are relevant to their lives positively affected students' attitudes

Using authentic problems solving to teach mathematics and its effect on students' attitude was the focus of a study conducted by Olson (1998). Olson stated that "Seventy-five percent of the students possess positive or very positive attitudes towards math. Of the remaining 25%, only 10% classify their attitude as negative. None of the students feel very negative towards math" (p. 31).The findings of this dissertation agree with the findings of Olson (1998). When asked about the change of their attitude after the study, 51%of the control group and 61% of the experimental group indicated that their attitude improved. From the control group, 15% indicated their attitude became worse versus 5% from the experimental group. In the control group, 22% of the students indicated that their attitude did not change versus 14% in the experimental group. The rest of the students, 12% in the control group and 16% in the experimental group, gave no answer.

Using authentic problem solving positively affected students' attitude and achievement was the finding of a study conducted by Devens-Seligman (2007) and a study conducted by Middleton and Spanius (1999). Devens-Seligman indicated that "for the students in this study, both their attitudes and understanding of mathematics improved as a result of solving non-routine problems" (p. 113).According to Middleton and Spanius (1999),motivation is positively associated with achievement, they indicated that "achievement motivation in mathematics is highly influenced by instructional practices, and if appropriate practices are consistent over a long period of time, children can and do learn to enjoy and value mathematics (p.13).

The findings of this dissertation agree with the findings of Devens-Seligman (2007) and Middleton and Spanius (1999). The qualitative and the quantitative data indicated that the improvement in achievement and attitude in the experimental group was more than the control group.

Limitations of the Study

This study revealed that using authentic problem solving in a group setting helped students improve their conceptual understanding, which enabled them to answer more in-depth questions. The group that used authentic problem solving achieved more than the group who used the traditional curriculum. The qualitative data revealed that using authentic problem solving improved students' attitude more than when the traditional curriculum was used. However, this study has six limitations related to the students, the teachers and the time in which the study was conducted.

The first limitation is the fact that this study was done in two charter schools in which most parents were involved and cared about their children's education. Parents in both schools kept up with their children's grades, made sure homework was done daily, and were in touch with the teachers. This might not be the case in all schools; involving more schools from different areas would make the results of this study more valid.

The second limitation of this study is that the study used a total of 85 students: 41 from one school and 44 from the other school. These students were advanced students who excelled in mathematics. The school used an accelerated curriculum that challenged them since fifth grade. There were no failing students in these classes, but some students were struggling. This is not always the case in Algebra I classes. If the study was conducted in a regular class, the classroom would have a wider range of student abilities.

The third limitation of the study is that the study took place ten days before winter break. This left no time for extension if needed. The students in both groups would have benefitted if the study was extended. The beginning of the unit was a review of simplifying exponents; both groups did not learn it in their previous year because it was at the end of the book, and they did not have time. The teachers had to take time to teach it, which left less time to complete the rest of the unit. Toward the end of the study, some students took a couple days off before winter break, during which the post-test was given. These students had to take the test after break. By that time, they forgot a lot of the material. The final few days of the study had a lot of assessment for both knowledge and attitude. This might have caused some students to be exhausted and to not care because they knew that most of the assessments would not be counted in their class grade.

The fourth limitation is that the study was conducted in two different schools with two different teachers. The teachers' personalities and their ways of interacting with the students were completely different. When students took the pre-attitude survey, the first teacher told the students that the survey will not be viewed by the teacher or anyone from the school so the need to write their honest opinion. The second teacher did not say anything before passing the survey to the students. This might have given them the impression that the teacher would see their answers so they did not write their honest opinion. The difference between the two teachers in the teaching style, in checking homework, and in checking for students' understanding varied greatly. It would have been better to have the same teacher teach both groups to eliminate the difference between the teachers.

The fifth limitation was the way some of the data was collected in this study. Students self-reported their attitude in the pre- and post-surveys. They self-evaluated each of the questions without having to provide any explanation of their choices. Students indicated that their attitude improved, but many of them never indicated why.

The last limitation was the interpretation of problem solving between the two groups. The curriculums were different, so the meaning of problem solving was interpreted differently by the two groups. The control group was given some problem solving at the end of the lesson as well as at the end of their homework; the numbers were clearly identifiable and could easily be substituted in the formula. That was the only type of problem solving they encountered. The experimental group used authentic problem solving in which they had to try multiple methods in order to solve a given problem with a real-world context. Each group interpreted problem solving according to their experience. When the students in the control group indicated that their problem solving improved they referred to the problems they were exposed to. These problems were different than the problems the experimental group experienced.

Implications for other Teachers and Researchers

The purpose of this study was to determine the effect of using authentic problem solving on students' achievement and attitude. The quantitative and qualitative data revealed that the experimental group achieved more than the control group and their attitude was positively affected more than the control group. This study will add additional evidence to the research that using problem solving improves students' achievement and attitude.

Most of the students who learn to carry out procedures without adequate focus on the concept will eventually forget the procedures. If the purpose of learning mathematics is to equip students with the tools to help them solve problems, then focusing only on the procedure is not the best way to teach mathematics. Students must understand and make sense of the concept along with procedures in order to solve the problems they will face in their future. The choice of the curriculum is very important on students' achievement and attitude. For the teachers who have a curriculum that focuses only in procedural knowledge, they need to supplement it with a lot of problem solving to help students build their conceptual understanding. For the teachers who have a curriculum that is focused on conceptual understanding, they need to supplement it with procedural knowledge.

Teachers should move away from the traditional teaching method. They need to help students arrive to or even discover how to arrive to the solution instead of giving them a formula to substitute the numbers in to solve. They also need to give students the opportunity to discuss the problems in groups in addition to individual work. Instead of requiring students to copy notes from the board, the teacher needs to help guide them to build their understanding of the concept by spending more time on solving problems that are relevant to their lives.

For schools and administrators, this research revealed the effect the curriculum can have on students. It is necessary to provide students with a curriculum that helps prepare them for the future. The 21st century requires interacting with other people and collaborating to solve a given problem, not just following a given procedure. The reformed approach in teaching mathematics is far more effective in helping prepare students for future challenges.

This research was done on a sample of Algebra 1 students. Despite the limitations of the study, we can conclude that, when students are actively involved in their learning, they achieve more. We can also conclude that using problem solving to help students construct their conceptual understanding improves students' attitudes and articulation of their attitudes more than using a curriculum that focuses on procedural knowledge.

Questions/Problems for Further Study

The role of determining the effect of any curriculum on achievement and attitude lies on the shoulders of researchers. Examining the effectiveness of the mathematics curriculums is an essential task to help schools choose the best curriculum that will prepare students for future challenges. It is essential to conduct studies to determine the effectiveness of the mathematics curriculum as well as to decide if it really prepares the students for the future. This study examined the effect on students' attitude and achievement when authentic problem solving is used in order to teach exponential functions.

In this study, the two groups were taught by two different teachers, one question comes to mind is that whether the achievement gap and the difference in attitude would be different if the same teacher taught both groups?

Moreover, the differences in the curriculum trigger another question. If the traditional group supplemented their curriculum with problem solving, and the experimental supplemented their curriculum with procedural knowledge, how would the achievement be different? Would students' attitudes between the two groups be different?

Students in the experimental group used group work while solving authentic problems. Did the experimental group do better on the post test and other assessments due to the authentic problem solving approach, the group work, or the combination of the two? Did the experimental group improve their attitude due to the authentic problem solving approach, the group work, or the combination of the two?

This study lasted for ten day. Would the achievement gap between the two groups be different if the study took longer? What would the results look like if the study was done in more than two schools and more than one unit in Algebra 1?

This study could be extended in order to determine the effect of teaching using authentic problem solving on other mathematics classes such as Geometry or Probability and Statistics.

APPENDIX A

Treatment group lesson plans (Total of 10 days)

Investigation 1 (2 days)

Objectives

• Gain an intuitive understanding of basic exponential growth patterns

• Begin to recognize exponential patterns in tables, graphs, and equations

• Solve problems involving exponential growth

• Understand the role of the growth factor in exponential relationships

• Express a product of identical factors in both exponential form and standard form

• Write equations for exponential relationships represented by tables and graphs

• Make a table from the graph of an exponential relationship

• Compare different exponential growth patterns and compare exponential and linear growth

Benchmarks

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Describe qualitatively the functional relationship between two quantities by analyzing a graph e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Learning Resources and Materials

Notebook, graph paper, and graphing calculator

Development of Lesson

This investigation is divided into four sections each of which includes a problem with a set of questions to guide students' work and exploration along with a follow up problem. The follow up problem consists of multiple questions to help extend students' understanding. The homework is an extension to the problem they investigated.

Problem 1.1: Students start by investigating the growth in the number of ballots created by repeatedly cutting a piece of paper in half. They will make a table and a graph that relates the number of cuts and the number of ballots. Students need to find a pattern between the number of cuts and the number of ballots to help them predict how many ballots will result in 20, 40, and 50 cuts. Homework: 1–4, 31.

Problem 1.2: Students investigate an exponential situation set in the fictitious ancient kingdom of Montarek. A requested reward calls to place one coin on the first square of a chessboard, two on the second square, four on the third square, and so on. Students explore patterns of change in this exponential relationship. Homework: 5–21, 32, 33, 39–46

Problem 1.3: Students consider two variations on the previous problem. In the first, the number of coins is tripled on each square. In the second, the number is quadrupled. Students make tables and graphs for the variations, describe their patterns, and write equations for them. The focus is looking for a general form for exponential equations. The term *growth factor* is introduced. Homework: 22, 23, 34, 47–49

Problem 1.4: Students compare a linear relationship to exponential relationships. They have to decide when each relationship is better for the reward recipient and why. Homework: 24–30, 35–38, 50

Introduction

This investigation uses the constructivism teaching method. The teacher job is to guide students' learning not to provide the information to students. The students are actively involved in building their new knowledge to fit with their prior knowledge.

Methods/Procedures

Group constructivism teaching method is what will be used for this investigation. The teacher facilitates and helps each group by asking questions to help guide students' work. Each investigation is a set of questions they have to investigate.

•The essential questions are how to recognize exponential growth patterns and how to solve problems involving exponential growth patterns.

Accommodations/Adaptations

Group constructivism teaching method is a differentiation method that meets students' needs because each student builds his/her understanding in his/her own way according to their prior knowledge. The teacher must ask questions to all students individually and as groups to make sure they are building a correct understanding. The teacher needs to set up group work rules to make sure that everybody is participating in the work.

Assessment/Evaluation

In group constructivism teaching method assessment/evaluation is an on-going process because the teacher is continuously facilitating the learning of the students, evaluating their work, giving feedback, and correcting their misunderstandings.

Closure

At the end of the investigation, the class as a whole discusses their findings; each group presents the way they solved the problem. By this time the teacher is well aware of the work of each group, has approved their answers, and has already corrected any misconceptions or misunderstandings.

Teacher Reflection

Investigation 2 (2 days)

Objectives

• Recognize exponential growth in verbal descriptions, tables, graphs, and equations

• Determine and interpret the *y*-intercept (initial value) for an exponential relationship

• Determine the growth factor based on a verbal description, table, graph, or equation for an exponential relationship

• Write an equation for an exponential relationship from its graph

• Solve problems involving exponents and exponential growth

Benchmarks

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Describe qualitatively the functional relationship between two quantities by analyzing a graph e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Learning Resources and Materials

Notebook, graph paper, and graphing calculator

Development of Lesson

This investigation is divided into three sections each of which is a problem with a set of questions to guide students work and exploration followed by a follow up problem, the follow up problem consists of multiple questions to help extend students' understanding. The homework is an extension to the problem they investigated.

Problem 2.1: Students read about a real situation in which a nonnative plant spread rapidly and began to cover Lake Victoria. The area of the plant doubles every ten days. Students then solve a problem about a similar situation in Ghost Lake. In both problems, the area of the plant doubles and the starting value is greater than 1. Students will need to make a table, write an equation to represent the situation, and graph it. Homework: 1–4, 15–21, 31–33

Problem 2.2: In this activity, students explore the exponential growth of mold. An equation for the growth pattern is given. Students find the initial amount of mold and the growth factor from the equation. They then use the equation to determine the y-intercept, find the growth factor, and answer specific questions about the situation. Homework: 5–8, 22, 23

Problem 2.3: Exponential data is presented in the form of a graph. Students find and interpret the *y*-intercept and growth factor then use this information to write an equation. Students use the graph or equation to answer questions about the situation. Homework: 9–14, 24–30

Introduction

This investigation uses the constructivism teaching method. The teacher job is to guide students' learning not to provide the information to students. The students are actively involved in building their new knowledge to fit with their prior knowledge.

Methods/Procedures

Group constructivism teaching method is what will be used for this investigation. The teacher facilitates and helps each group by asking questions to help guide students' work. Each investigation is a set of questions they have to investigate.

•The essential questions are how to recognize exponential growth patterns and how to solve problems involving exponential growth patterns.

Accommodations/Adaptations

Group constructivism teaching method is a differentiation method that meets students' needs because each student builds his/her understanding in his/her own way according to their prior knowledge. The teacher must ask questions to all students individually and as groups to make sure they are building a correct understanding. The teacher needs to set up group work rules to make sure that everybody is participating in the work.

Assessment/Evaluation

In group constructivism teaching method assessment/evaluation is an on-going process because the teacher is continuously facilitating the learning of the students, evaluating their work, giving feedback, and correcting their misunderstandings.

Closure

At the end of the investigation, the class as a whole discusses their findings; each group presents the way they solved the problem. By this time the teacher is well aware of the work of each group, has approved their answers, and has already corrected any misconceptions or misunderstandings.

Teacher Reflection

Investigation 3 (1.5 days)

Objectives

- Determine a non-whole number growth factor using information in a table or a graph.
- Determine the growth rate, or percent change.
- Use sample population data to write an equation to model population growth.
- Investigate the growth of an investment with a given growth rate (percent increase).
- Relate growth rates and growth factors.
- Review and extend understanding of percent.
- Understand the role of initial value (*y*-intercept) in compound growth.

Benchmarks

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear

Describe qualitatively the functional relationship between two quantities by analyzing a graph e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Learning Resources and Materials

Notebook, graph paper, and graphing calculator

Development of Lesson

This investigation is divided into three sections each of which is a problem with a set of questions to guide students work and exploration followed by a follow up problem, the follow up problem consists of multiple questions to help extend students' understanding. The homework is an extension to the problem they investigated.

Problem 3.1: This problem is set in the context of a historical account of a rapidly multiplying rabbit population in Australia. Students use population data given in table form to write an equation to model population growth. To find the overall growth factor, students need to find the growth factor for consecutive years and decide on a typical, or average, value. The growth factor in this situation is not a whole number. Students also are asked to find the doubling time for a population. Homework: 1–8, 24–30

Problem 3.2: Students will examine patterns of change due to compound growth in the value of stamp collection. Students look at the growth in the value of a stamp collection for 6% and 4% annual rates of growth. They use these rates to find the growth factors Students learn the connection between growth rate (or percent change) and growth factor. Homework: 9–20, 31, 32, 40–45

Problem 3.3: Students study the effects of the initial value (y-intercept) on the growth patterns of three different savings plans. In this problem, they use growth rates and different starting values to write equations for three different savings plans set up by a grandmother for her grandchildren. Homework: 21–23, 33–39, 46, 47

Introduction

This investigation uses the constructivism teaching method. The teacher job is to guide students' learning not to provide the information to students. The students are actively involved in building their new knowledge to fit with their prior knowledge.

Methods/Procedures

Group constructivism teaching method is what will be used for this investigation. The teacher facilitates and helps each group by asking questions to help guide students' work. Each investigation is a set of questions they have to investigate.

The essential questions are how to recognize exponential growth patterns and how to solve problems involving exponential growth patterns.

Accommodations/Adaptations

Group constructivism teaching method is a differentiation method that meets students' needs because each student builds his/her understanding in his/her own way according to their prior knowledge. The teacher must ask questions to all students individually and as groups to make sure they are building a correct understanding. The teacher needs to set up group work rules to make sure that everybody is participating in the work.

Assessment/Evaluation

In group constructivism teaching method assessment/evaluation is an on-going process because the teacher is continuously facilitating the learning of the students, evaluating their work, giving feedback, and correcting their misunderstandings.

Closure

At the end of the investigation, the class as a whole discusses their findings; each group presents the way they solved the problem. By this time the teacher is well aware of the work of each group, has approved their answers, and has already corrected any misconceptions or misunderstandings.

Teacher Reflection

Investigation 4 (2.5 days)

Objectives

• Use knowledge of exponential relationships to make tables and graphs and to write equations for exponential decay patterns

• Analyze and solve problems involving exponents and exponential decay

• Recognize patterns of exponential decay in tables, graphs, and equations

• Use information in a table or graph of an exponential relationship to write an equation

• Analyze an exponential decay relationship that is represented by an equation and use the equation to make a table and graph

Benchmarks

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Describe qualitatively the functional relationship between two quantities by analyzing a graph e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features

of a function that has been described verbally.

Learning Resources and Materials

Notebook, graph paper, and graphing calculator

Development of Lesson

This investigation is divided into three sections each of which is a problem with a set of questions to guide students work and exploration followed by a follow up problem, the follow up problem consists of multiple questions to help extend students' understanding. The homework is an extension to the problem they investigated.

Problem 4.1: Students will revisit the paper-cutting activity of Investigation 1 with a new question in mind: How does the area of a ballot change with each successive cut? They are asked make a table, graph it, and write an equation to represent the situation. Homework: 1, 2, 8

Problem 4.2: For animals and human beings alike, many chemicals introduced into the bloodstream break down, or metabolize, in patterns that are modeled well by exponential decay. In this problem, students analyze the breakdown of a preventative flea medicine in a dog's blood, as represented in a table and a graph. Students find the decay factor associated with the data, use it to write an equation, and then consider the role of the initial dose and its effect on the equation. They are then given the initial dose and *decay rate* for a similar situation, and use this information to make a table and write an equation. They also investigate the connection between decay rate and decay factor.. Homework: 3–5, 9–11, 13

Problem 4.3: Students will conduct an experiment to determine the rate at which a cup of water cools, a phenomenon that can be closely modeled by exponential decay. Homework: 6, 7, 12

Introduction

This investigation uses the constructivism teaching method. The teacher job is to guide students' learning not to provide the information to students. The students are actively involved in building their new knowledge to fit with their prior knowledge.

Methods/Procedures

Group constructivism teaching method is what will be used for this investigation. The teacher facilitates and helps each group by asking questions to help guide students' work. Each investigation is a set of questions they have to investigate.

The essential questions are how to recognize exponential growth patterns and how to solve problems involving exponential growth patterns.

Accommodations/Adaptations

Group constructivism teaching method is a differentiation method that meets students' needs because each student builds his/her understanding in his/her own way according to their prior knowledge. The teacher must ask questions to all students individually and as groups to make sure they are building a correct understanding. The teacher needs to set up group work rules to make sure that everybody is participating in the work.

Assessment/Evaluation

In group constructivism teaching method assessment/evaluation is an on-going process because the teacher is continuously facilitating the learning of the students, evaluating their work, giving feedback, and correcting their misunderstandings.

Closure

At the end of the investigation, the class as a whole discusses their findings; each group presents the way they solved the problem. By this time the teacher is well aware of the work of each group, has approved their answers, and has already corrected any misconceptions or misunderstandings.

Teacher Reflection

Investigation 5 (2 days)

Objectives

- Examine patterns in the exponential and standard forms of powers of whole numbers
- Use patterns in powers to estimate the ones digits for unknown powers
- Use patterns in powers to develop rules for operating with exponents
- Become skillful in operating with exponents in numeric and algebraic expressions
- Describe how varying the values of *a* and *b* in the equation $y = a(bx)$ affects the graph of that equation

Benchmarks

Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Learning Resources and Materials

Notebook, graph paper, and graphing calculator

Development of Lesson

This investigation is divided into three sections each of which is a problem with a set of questions to guide students work and exploration followed by a follow up problem, the follow up problem consists of multiple questions to help extend students' understanding. The homework is an extension to the problem they investigated.

Problem 5.1: Students examine patterns in the ones digits of powers. They use these patterns to predict ones digits for powers that would be tedious to find directly. Students create a powers table for a^m for a = 1 to a = 10 and m = 1 to m = 8.They use the table to find patterns in order to predict the ones digits for powers such as 2^{100}, 6^{50}, and 17^{20}, and to estimate the standard form of these powers. Homework: 1–9, 44, 45, 52–55

Problem 5.2: Students use their work in the previous problem to develop rules for operating with numerical expressions with exponents. For example students use the powers table from Problem 5.1 to find special relationships among the product of numbers written in exponential form, and then they will conclude all the properties of exponents. Homework: 10–41, 46–48, 56–63

Problem 5.3: Students use graphing calculators to study how the values of a and b affect the graph of $y = a(b^x)$. Homework: 42, 43, 49–51

Introduction

This investigation uses the constructivism teaching method. The teacher job is to guide students' learning not to provide the information to students. The students are actively involved in building their new knowledge to fit with their prior knowledge.

Methods/Procedures

Group constructivism teaching method is what will be used for this investigation. The teacher facilitates and helps each group by asking questions to help guide students' work. Each investigation is a set of questions they have to investigate.

The essential questions are how to recognize exponential growth patterns and how to solve problems involving exponential growth patterns.

Accommodations/Adaptations

Group constructivism teaching method is a differentiation method that meets students' needs because each student builds his/her understanding in his/her own way according to their prior knowledge. The teacher must ask questions to all students individually and as groups to make sure they are building a correct understanding. The teacher needs to set up group work rules to make sure that everybody is participating in the work.

Assessment/Evaluation

In group constructivism teaching method assessment/evaluation is an on-going process because the teacher is continuously facilitating the learning of the students, evaluating their work, giving feedback, and correcting their misunderstandings.

Closure

At the end of the investigation, the class as a whole discusses their findings; each group presents the way they solved the problem. By this time the teacher is well aware of the work of each group, has approved their answers, and has already corrected any misconceptions or misunderstandings.

Teacher Reflection

APPENDIX B

Control group lesson plans (Total of 10 days)

Lesson 1 (1 day)

Content

1. Use properties of exponents to multiply exponential expressions.

2. Use powers to model real-life problems.

Benchmarks

Work with radicals and integer exponents

Learning Resources and Materials

Paper, pencil, scientific calculator, and white board for students. A computer with a projector for the teacher.

Development of Lesson

The lesson is divided into three sections. The first section is the product of power property, the second section is the power of power property, and the last section is the power of product property. For the first two sections the teacher will start by giving the main rule, the new vocabulary, and an example to clarify the rule. In the third section the teacher will explain how to use the first two properties to come up with the third property. In each section the teacher will guide the students through four examples then give students four similar questions to answer on their own. The last section includes an extra two examples. One shows students how to use all learned properties together to simplify exponential expressions, and the other is a real world problem in which the students have to use one of the properties.

Homework

The book has 59 problems as exercises for the lesson. 51 of them are direct applications of the properties, the last seven are labeled problem solving, and the last problem is labeled challenging.

Introduction

Before the lesson, students will finish a worksheet labeled investigating activity 8.1. In this activity students will fill in a provided table by multiplying numbers in exponential form as an introduction to using the multiplication property of exponents.

Methods/Procedures

•Teacher guided instructions using a power point presentation

• Students work individually

•The essential question to guide and focus the teaching and learning is "How to use multiplication properties of exponents to evaluate and simplify expressions."

Accommodations/Adaptations

The lesson will be chunked into multiple sections; each section is also chunked into parts that increase in the level of complexity. Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Using the power point as a visual representation, demonstrating the solutions on the board, and explaining the concept in multiple ways are what help the teacher reach out to students with all learning styles.

At the end of the lesson the teacher gives an exit ticket that is two or three questions long to do a final check for understanding. As the students finish solving the questions, they turn them in for immediate feedback. This will allow the teacher to identify who needs more help.

Assessment/Evaluation

Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Exit tickets and entry tickets are used as a formative assessment to help the teacher determine if students understood the concept. Students check homework from the correct answers that are posted, correct their mistakes, and ask questions if they don't understand their mistakes. Homework is collected and a few problems are checked by the teacher to make sure that the students understood the lesson.

Closure

After the exit ticket the teacher will decide whether to revisit the topic the next day or move on to the next lesson.

Teacher Reflection

Lesson 2 (1day)

Content

1. Evaluate powers that have zero and negative exponents.

2. Graph exponential functions.

Benchmarks

Work with radicals and integer exponents

Learning Resources and Materials

Paper, pencil, scientific calculator, white board, and a computer with a projector for the teacher.

Development of Lesson

The lesson is divided into three sections: the first is the quotient of powers property, the second is the power of quotient property, and the last section is solving multi-step problems. In the first two sections, the teacher will guide the students through four examples then give students four similar questions to answer on their own. The third section consists of two examples (fractals trees and real-world problems), and two guided practice problems that are identical to the examples but with different numbers.

Homework

48 direct applications of the properties along with a problem solving section with 6 problems.

Introduction

Check homework

Change some variables or numbers from two or three problems from the homework and ask students to answer them on their whit boards

Methods/Procedures

•teacher guided instructions using power point presentation

• individualized

•The essential question to guide and focus the teaching and learning is: How to evaluate and simplify powers that have zero and negative exponents.

Methods/Procedures

•Teacher guided instructions using a power point presentation

• Students work individually

•The essential question to guide and focus the teaching and learning is "How to use multiplication properties of exponents to evaluate and simplify expressions."

Accommodations/Adaptations

The lesson will be chunked into multiple sections; each section is also chunked into parts that increase in the level of complexity. Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Using the power point as a visual representation, demonstrating the solutions on the board, and explaining the concept in multiple ways are what help the teacher reach out to students with all learning styles.

At the end of the lesson the teacher gives an exit ticket that is two or three questions long to do a final check for understanding. As the students finish solving the questions, they turn them in for immediate feedback. This will allow the teacher to identify who needs more help.

Assessment/Evaluation

Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Exit tickets and entry tickets are used as a formative assessment to help the teacher determine if students understood the concept. Students check homework from the correct answers that are posted, correct their mistakes, and ask questions if they don't understand their mistakes. Homework is collected and a few problems are checked by the teacher to make sure that the students understood the lesson.

Closure

After the exit ticket the teacher will decide whether to revisit the topic the next day or move on to the next lesson.

Teacher Reflection

Lesson 3 and extension (2days)

Content

1. Use the division properties of exponents to evaluate powers and to simplify expressions.
2. Use the division property of exponents to solve real-life problems.

Benchmarks

Work with radicals and integer exponents

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})3 = (5^{1/3})^3$ to hold, so $(5^{1/3})^3$ must equal 5.

Rewrite expressions involving radicals and rational exponents using the properties of exponents

Learning Resources and Materials

Paper, pencil, scientific calculator, white board, and a computer with a projector for the teacher.

Development of Lesson

The lesson is divided into two sections; the first of which is the definition and the simplification of zero and negative exponents. The second section is to review all the properties and to simplify exponential expressions. The teacher will explain how to simplify exponential expressions using four examples, and then students will individually solve four similar problems on their own. At the end of the two sections an extra two examples will be given. The first is to simplify an exponential expression using more than one property, and the second is a standardized test practice question.

Homework

49 direct application questions of the properties along with a problem solving section with 9 problems with a challenge problem at the end.

Lesson 3 Extension (1 day)

After this lesson, students will be introduced to fractional exponents to relate square and cubic roots to exponents to the power of 1/2 or 1/3. The teacher will explain square roots using four problems then explain cubic roots using four problems. Finally, the properties of exponents will be used to simplify exponents to the power of 1/2 or 1/3. 13 problems will be given to the students as practice.

Homework

6 review problems from the previous sections.

Introduction

Check homework

Change some variables or numbers from two or three problems from the homework and ask students to answer the questions on their whit boards.

Before the lesson students will finish a worksheet labeled investigating activity 8.3. In this investigation students will fill in a provided table by evaluating 2^n and 3^n for a given n. This will help them start thinking about how to simplify expressions with zero or negative exponents.

Methods/Procedures

•Teacher guided instructions using a power point presentation

• Students work individually

•The essential question to guide and focus the teaching and learning is "How to use multiplication properties of exponents to evaluate and simplify expressions."

Accommodations/Adaptations

The lesson will be chunked into multiple sections; each section is also chunked into parts that increase in the level of complexity. Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Using the power point as a visual representation, demonstrating the solutions on the board, and explaining the concept in multiple ways are what help the teacher reach out to students with all learning styles.

At the end of the lesson the teacher gives an exit ticket that is two or three questions long to do a final check for understanding. As the students finish solving the questions, they turn them in for immediate feedback. This will allow the teacher to identify who needs more help.

Assessment/Evaluation

Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Exit tickets and entry tickets are used as a formative assessment to help the teacher determine if students understood the concept. Students check homework from the correct answers that are posted, correct their mistakes, and ask questions if they don’t understand their mistakes. Homework is collected and a few problems are checked by the teacher to make sure that the students understood the lesson.

Closure

After the exit ticket the teacher will decide whether to revisit the topic the next day or move on to the next lesson.

Teacher Reflection

Lesson 4 (1 day)

Content

1. Use scientific notation to represent numbers.
2. Use scientific notation to describe real-life situations.

Benchmarks

CCSS. 8.EE. 4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology

Learning Resources and Materials

Paper, pencil, scientific calculator, white board, and a computer with a projector for the teacher.

Development of Lesson

Students learned how to convert from scientific notation to standard notation and vise versa in 6^{th} and 7^{th} grade. The lesson is divided into five sections. The first section starts by giving two problems on how to convert from standard notation to scientific. The first example uses a very large number, and the second example uses a very small decimal. The second section consists of two problems to convert from scientific notation to standard notation; the first problem has a positive exponent while the second problem has a negative exponent. Students will do two problems on their own. The third section is about how to order numbers written in scientific notation. The fourth section is about computation with scientific notation; the teacher will demonstrate how to apply the product of power property, the power of product property, and the quotient of power property to numbers written in scientific notation. Students have four problems to practice in the last two sections. The final section is a problem solving application in which the teacher will demonstrate how to apply what was learned to solve problems. Students have one question to do on their own which requires them to solve the example problem but with different numbers.

Homework:

50 direct application problems with the last one labeled challenging along with 10 problem solving problems.

Introduction

Check homework

Change some variables or numbers from two or three problems from the homework and ask students to answer the questions on their whit boards.

Methods/Procedures

•Teacher guided instructions using a power point presentation

• Students work individually

•The essential question to guide and focus the teaching and learning is “How to use multiplication properties of exponents to evaluate and simplify expressions.”

Accommodations/Adaptations

The lesson will be chunked into multiple sections; each section is also chunked into parts that increase in the level of complexity. Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Using the power point as a visual representation, demonstrating the solutions on the board, and explaining the concept in multiple ways are what help the teacher reach out to students with all learning styles.

At the end of the lesson the teacher gives an exit ticket that is two or three questions long to do a final check for understanding. As the students finish solving the questions, they turn them in for immediate feedback. This will allow the teacher to identify who needs more help.

Assessment/Evaluation

Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Exit tickets and entry tickets are used as a formative assessment to help the teacher determine if students understood the concept. Students check homework from the correct answers that are posted, correct their mistakes, and ask questions if they don’t understand their mistakes. Homework is collected and a few problems are checked by the teacher to make sure that the students understood the lesson.

Closure

After the exit ticket the teacher will decide whether to revisit the topic the next day or move on to the next lesson.

Teacher Reflection

Lesson 5 (2 days)

Content

1. Write and use models for exponential growth.

2. Graph models for exponential growth.

Benchmarks

CCSS. A-CED1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

F-BF1 : Write a function that describes a relationship between two quantities

F-LE1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

Learning Resources and Materials

Paper, pencil, graph paper, scientific calculator, white board, and a computer with a projector for the teacher.

Development of Lesson

The lesson is divided into five sections. The first section compares two tables and their equations; one table represents a linear growth while the other represents an exponential growth. The teacher explained that in linear growth, a certain number is added while in exponential growth a certain number is multiplied. This number is labeled "b" in the exponential equation which is of the form $y= ab^x$ while "a" is the number you get when you substitute zero for x. The teacher demonstrated how to write the equation of an exponential function if the table is given. The students have one table to practice on their own. The second section of the lesson is graphing exponential functions. The teacher demonstrated how to make a table and graph $y= 2^x$.

The third section is about comparing graphs of exponential functions; the teacher made a table with four columns labeled: x, 2^x, $3(2^x)$, and $-3(2^x)$. The teacher then demonstrated how to graph the three functions on the same set of axis. The teacher then explained how changing "a" in the exponential function equation affects the graph. The students practiced three graphing and comparing problems on their own. The fourth section is the exponential growth model in which the teacher gave the exponential growth model [$y = a(1+r)^t$, a is the initial value,1+r is the growth factor, r is the growth rate, and t is the time period]. The teacher then demonstrated how to use the formula to write an exponential growth model to represent a given situation. The final section explains that compound interest is actually an exponential growth model. The teacher demonstrated how to calculate the value of an investment earning a yearly compound interest. Students practice on their own by doing the last two examples but with different numbers.

Homework

35 direct application problems, two problems labeled challenging, and 14 problem solving questions with the last one labeled challenging

Introduction

Check homework

Change some variables or numbers from two or three problems from the homework and ask students to answer the questions on their whit boards.

Methods/Procedures

•Teacher guided instructions using a power point presentation

• Students work individually

•The essential question to guide and focus the teaching and learning is "How to use multiplication properties of exponents to evaluate and simplify expressions."

Accommodations/Adaptations

The lesson will be chunked into multiple sections; each section is also chunked into parts that increase in the level of complexity. Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Using the power point as a visual representation, demonstrating the solutions on the board, and explaining the concept in multiple ways are what help the teacher reach out to students with all learning styles.

At the end of the lesson the teacher gives an exit ticket that is two or three questions long to do a final check for understanding. As the students finish solving the questions, they turn them in for immediate feedback. This will allow the teacher to identify who needs more help.

Assessment/Evaluation

Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Exit tickets and entry tickets are used as a formative assessment to help the teacher determine if students understood the concept. Students check homework from the correct answers that are posted, correct their mistakes, and ask questions if they don’t understand their mistakes. Homework is collected and a few problems are checked by the teacher to make sure that the students understood the lesson.

Closure

After the exit ticket the teacher will decide whether to revisit the topic the next day or move on to the next lesson.

Teacher Reflection

Lesson 6 (2 days)

Content

1. Write and use models for exponential decay.
2. Graph models for exponential decay.

Benchmarks

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Write a function that describes a relationship between two quantities

Distinguish between situations that can be modeled with linear functions and with exponential functions.

Learning Resources and Materials

Paper, pencil, scientific calculator, white board, and a computer with a projector for the teacher.

Development of Lesson

The lesson is divided into five sections. The first section is about how to write exponential functions for a given table when b<1. The teacher will demonstrate how to write the exponential function for two functions; in the first one b>1 and in the second b<1. Students have one practice problem to do on their own. The second section of the lesson is graphing exponential functions (with b<1). The teacher will demonstrate how to make a table and graph $y= (1/2)^x$. The third section compares graphs of exponential functions. The teacher will make a table with four columns: x, $(1/2)^x$, $3(1/2)^x$, and $-3(1/2)^x$. The teacher will then demonstrate how to graph the three functions on the same set of axis and explain how changing "a" in the exponential function equation affects the graph. The students will practice graphing and comparing two exponential functions on their own. The fourth section explains how to classify and write rules for a given graph. The teacher will demonstrate how to determine whether a function is a growth or decay and how to use points on the graph to write the function rule. Students have one practice problem in which they have to write its equation on their own. The fifth section is about the exponential decay model. The teacher will give the exponential decay model [$y= a(1-r)^t$, a is the initial value,1- r is the decay factor, r is the decay rate, and t is the time period] and demonstrate how to use the formula to write an exponential decay model to represent a given situation. Students practice one problem on their own by re-doing the last example but with different numbers.

Homework

46 problems. 42 of them are direct application and four of them are labeled challenging. The problem solving section has seven problems; one of them is labeled challenging.

Introduction

Check homework

Change some variables or numbers from two or three problems from the homework and ask students to answer the questions on their whit boards.

Methods/Procedures

•Teacher guided instructions using a power point presentation

• Students work individually

•The essential question to guide and focus the teaching and learning is "How to use multiplication properties of exponents to evaluate and simplify expressions."

Accommodations/Adaptations

The lesson will be chunked into multiple sections; each section is also chunked into parts that increase in the level of complexity. Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Using the power point as a visual representation, demonstrating the solutions on the board, and explaining the concept in multiple ways are what help the teacher reach out to students with all learning styles.

At the end of the lesson the teacher gives an exit ticket that is two or three questions long to do a final check for understanding. As the students finish solving the questions, they turn them in for immediate feedback. This will allow the teacher to identify who needs more help.

Assessment/Evaluation

Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Exit tickets and entry tickets are used as a formative assessment to help the teacher determine if students understood the concept. Students check homework from the correct answers that are posted, correct their mistakes, and ask questions if they don't understand their mistakes. Homework is collected and a few problems are checked by the teacher to make sure that the students understood the lesson.

Closure

After the exit ticket the teacher will decide whether to revisit the topic the next day or move on to the next lesson.

Teacher Reflection

Lesson 6 extension (1 day)

Content

Write and use models for geometric sequence.

Benchmarks

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Write a function that describes a relationship between two quantities

Distinguish between situations that can be modeled with linear functions and with exponential functions.

Learning Resources and Materials

Paper, pencil, scientific calculator, white board, and a computer with a projector for the teacher.

Development of lesson

Use the decay formula to write an exponential decay model to represent a given situation.

Students practice one problem on their own by re-doing the last example but with different numbers.

Introduction

Check homework

Change some variables or numbers from two or three problems from the homework and ask students to answer the questions on their whit boards.

Methods/Procedures

•Teacher guided instructions using a power point presentation

• Students work individually

•The essential question to guide and focus the teaching and learning is "How to use multiplication properties of exponents to evaluate and simplify expressions."

Accommodations/Adaptations

The lesson will be chunked into multiple sections; each section is also chunked into parts that increase in the level of complexity. Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Using the power point as a visual representation, demonstrating the solutions on the board, and explaining the concept in multiple ways are what help the teacher reach out to students with all learning styles.

At the end of the lesson the teacher gives an exit ticket that is two or three questions long to do a final check for understanding. As the students finish solving the questions, they turn them in for immediate feedback. This will allow the teacher to identify who needs more help.

Assessment/Evaluation

Using white boards to check for understanding gives the teacher a fast way to see who needs help and who understands it. Exit tickets and entry tickets are used as a formative assessment to help the teacher determine if students understood the concept. Students check homework from the correct answers that are posted, correct their mistakes, and ask questions if they don't understand their mistakes. Homework is collected and a few problems are checked by the teacher to make sure that the students understood the lesson.

Closure

After the exit ticket the teacher will decide whether to revisit the topic the next day or move on to the next lesson.

Teacher Reflection

APPENDIX C

Exponential functions pre-and post-test

Name _______________

1. Kai's ambition is to compete in a national bike race when he graduates from high school, but he will need to purchase a new racing bike by then. After a lot of research, he finds a bike that suits him. The bike costs $1,500. Over the summer, Kai raises $1,000 by doing odd jobs and collecting contributions from his family and friends. He invests the money in an account that pays 8% interest per year on the balance in the account.

Complete the table below to determine how long it will take Kai's account to be worth $1,500.

Year	Amount saved
0	$1,000
1	
2	
3	
4	
5	
6	

a) What is the growth factor?

b) Suppose that the cost of the bike rises by 4% per year. Complete the table below to show how the cost increases. There is no sales tax where Kai lives.

Year	Bike Cost
0	$1,500
1	
2	
3	
4	
5	
6	

c) What is the growth rate?

d) Graph the data sets in parts (a) and (b) on the grid.

Label each graph.

e) Look at your tables and graphs. Describe what they tell you about the situation.

f) Will Kai ever be able to buy the bike? Explain your answer.

2. Tribetts are fuzzy insects that reproduce at the rate of 50% every day. Suppose you begin with 100 tribetts.

a) Make a table showing the growth in the number of tribetts for the first 10 days.

b) On what day will their first be 1000 tribetts?

c) Write an equation for the relationship between days d and numbers of tribetts T.

3. The equation $y = 2(3^x)$ represent the growth pattern for a population of mice. Complete the following sentence by circling your choices. Your sentence should describe the pattern in words.

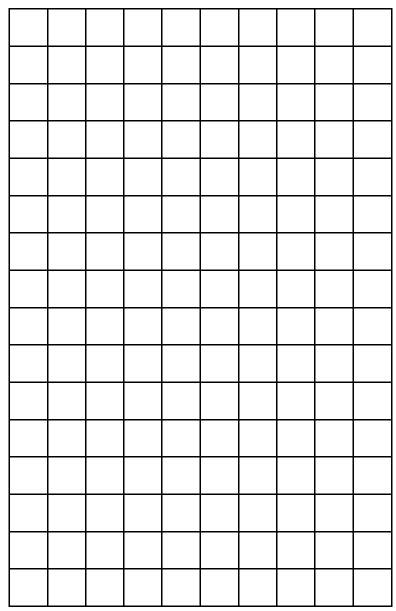
   a) The population started with ___________ mice.

   b) The population grew at a rate of ____________%__ .

4. You buy a mountain bike on layaway. The cost is $300, and the interest rate on what you owe is 1% per month.

   a. Fill in the table to find out what happens if you do not start paying for the bike for 6 months.

Month	Amount Owed
0	$300
1	
2	
3	
4	
5	
6	



   b. Make a graph showing how the amount you owe. Be sure to label the scales on the axes.

5. A bathtub is being filled at a rate of 2.5 gallons per minute. The bathtub will hold 20 gallons of water.

   a) How long will it take to fill the bathtub?

   b) Is the relationship described linear, exponential, or neither? Write an equation relating the variables.

6. An experimental plant has an unusual growth pattern. On each day, the plant doubles its height of the previous day. On the first day of the experiment, the plant grows to twice, or 2 times, its original height. On the second day, the plant grows to 4 times its original height. On the third day, the plant grows to 8 times its original height.
   Is the relationship described linear, exponential, or neither? Write an equation relating the variables.

7. Janelle deposits $2,000 in the bank. The bank will pay 5% interest per year, compounded annually.
   a) Write an equation for calculating the balance, *b*, at the end of any year *t*.

   b) Approximately how many years will it take for the original deposit to double in value? Explain your reasoning.

   c) Suppose the interest rate is 10%. Approximately how many years will it take for the original deposit to double in value? How does this interest rate compare with an interest rate of 5%?

8. A tree farm has begun to harvest a section of trees that was planted a number of years ago. The table shows the number of trees remaining for each of 8 years of harvesting.

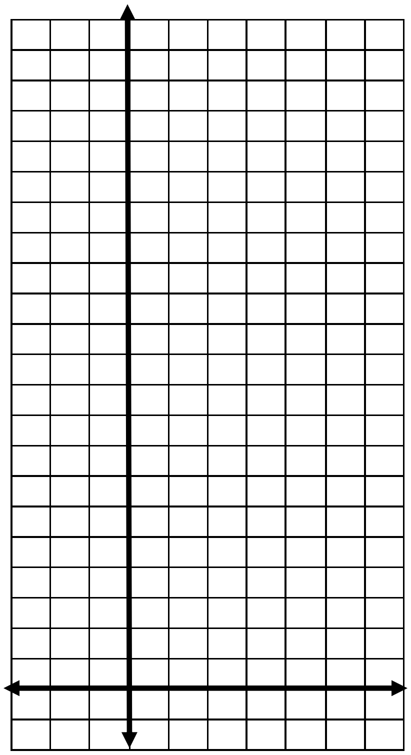
Year	0	1	2	3	4	5	6	7	8
Trees remaining	10,000	9502	9026	8574	8145	7737	7350	6892	6543

   a) Suppose the relationship between the year and the trees remaining are exponential. Approximate the growth or the decay factor for this relationship.

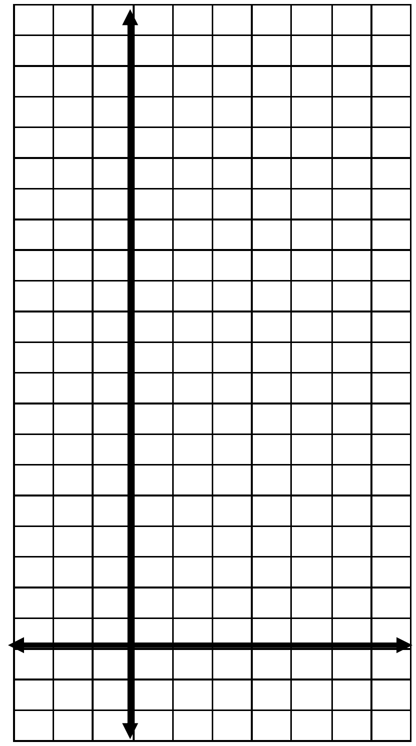
   b) Write an equation for the relationship between time and trees remaining.

9. On grid I, sketch and label graphs $y = 2^x$ and $y = 2.5^x$. On grid II, sketch and label graphs of $y = 0.5^x$ and $y = 0.2^x$.

Grid I



Grid II



a) In grid I, which equation represents the faster rate of growth?

b) In grid II, which equation represents the faster rate of decay?

c) How does the graph help you to answer parts (a) and (b)?

d) How do the equations help you to answer parts (a) and (b)?

APPENDIX D

Attitude Survey

Please answer all of the following questions as openly and as accurately as possible. Please rate how strongly you agree or disagree with each of the following statements by placing (X) under the appropriate response. *Answers will NOT affect your mathematics grade in any way.* Read each sentence carefully.

		Strongly agree	Agree	Neutral	Disagree	Strongly disagree
1	Mathematics helps develop the mind and teaches a person to think.					
2	Word problems in math class are unrealistic.					
3	Math class seems useless					
4	Math is essential to every career.					
5	Unless you are a math teacher, you will never use a lot of math from math class.					
6	Someday, I will get a job that does not use math.					
7	The job that I want to do will not use the math that I am learning in class.					
8	Mathematics is one of the most important subjects for people to study.					
9	I can think of many ways that I use math outside of school.					
10	Most jobs don’t need the math I am learning in class.					
11	Mathematics is important in everyday life.					
12	I often don’t know why we are learning what we are learning in math.					

Please rate how strongly you agree or disagree with each of the following statements by placing ((X) under the appropriate response. Read each sentence carefully as some sentences are positively stated while others are negatively stated.

		Strongly agree	Agree	Neutral	Disagree	Strongly disagree
13	Mathematics is one of my most dreaded subjects.					
14	My mind goes blank and I am unable to think clearly when working with mathematics.					
15	I am comfortable answering questions in math class.					
16	I am always under a terrible strain in a math class.					
17	I am happier in a math class than in any other class.					
18	Mathematics is a very interesting subject.					
19	It makes me nervous to even think about having to do a mathematics problem.					
20	I get a great deal of satisfaction out of solving a mathematics problem.					
21	Mathematics does not scare me at all.					
22	I feel a sense of insecurity when attempting mathematics					
23	I am able to solve mathematics problems without too much difficulty.					
24	Word problems seem like a foreign language to me.					
25	I like to solve new problems in mathematics.					
26	I really like mathematics.					
27	When I hear the word mathematics, I have a feeling of dislike.					
28	I am always confused in my mathematics class.					

29	I look for solutions to a difficult problem in math.					
30	I don't have a lot of self-confidence when it comes to mathematics.					
31	I believe I am good at solving math problems					
32	Mathematics makes me feel uncomfortable.					
33	Mathematics is dull and boring					
34	Studying mathematics makes me feel nervous.					

APPENDIX E

Control group entry tickets

Entry Ticket 1

Name___________________

Simplify. Write your answer as a power or as a product of powers.

1. $5^7 \cdot 5^4$

2. $(3^4)^5$

3. $(7m)^6$

4. $(-4x^7y^2)^3$

5. $[(5p+2)^2]^5$

6. $(\frac{2}{3}h^4)^3(5h^2)^4$

7. $(-6s^3t)^2(9s^2t^4)^3$

8. Simplify $(-b)^4 \cdot a^3 \cdot (b \cdot a)$ Evaluate the resulting expression when $a= -3$ and $b= -2$

9. Complete the statement using >or <
$(4^3 \cdot 5^2)^2$ _____ $4^9 \cdot 5^4$

Entry Ticket 2

Name____________________

Evaluate the exponential expression. Write fractions in simplest form.

1. $7(7^{-3})$

2. $(\frac{1}{3})^{-2}$

3. $(4^{-2})^2$

4. $12 \cdot 12^{-1}$

Rewrite the expression with positive exponents.

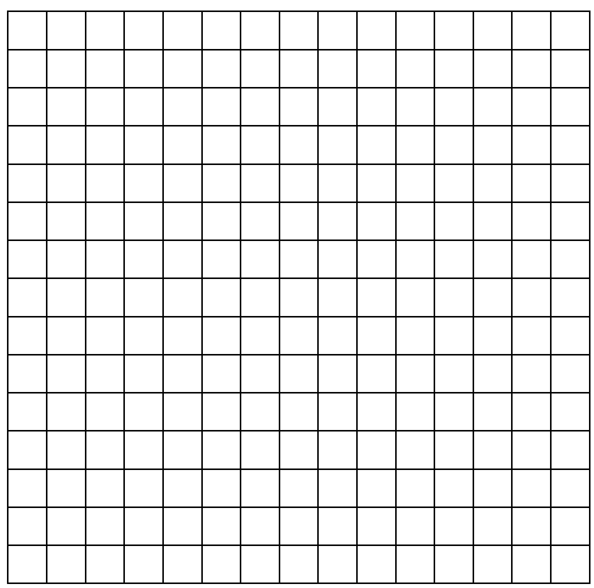
5. $m^{-5}n^5$

6. $\frac{1}{7k^{-3}}$

7. $(15r)^0$

8. . $\frac{2}{(8x)^{-3}}$

9. Graph the exponential function $y = (\frac{1}{7})^x$



Entry Ticket 3

Name___________________

Evaluate the expression. Write fractions in simplest form.

1. $\frac{7^5}{7^3}$

2. $(-\frac{2}{5})^4$

3. $(\frac{9}{4})^{-1}$

Simplify the expression. The simplified expression should have no negative exponents.

4. $b^7 \cdot \frac{b^4}{b^3}$

5. $\frac{-9m^4 n}{27mn^3}$

6. $\frac{-5x^{-3}y^4}{x^2y^{-1}} \frac{(3x^2y^2)^2}{x^3 y}$

7. The amount of money after *t* years in an account that begins with $700 and that earns compound interest at a rate of 4.7% per year can be modeled by the equation $A = 700(1.047)^t$ Find the ratio of the amount after 6 years to the amount after 4 years. Express the ratio as a power of 1.047.

Entry Ticket 4

Name___________________

Rewrite in decimal form.

1. 6.32×10^{-5} 2. 4.55×10^{0} 3. 7.168×10^{7}

Rewrite in scientific notation.

4. 3591.76

5. 0.0000849

6. 460,000,000

Evaluate the expression without using a calculator.

7. $(9\times10^{-12})\cdot(6\times10^{3})$

8. $\frac{1.8\times10^{-2}}{2.4\times10^{-7}}$

Entry Ticket 5

Name___________________

1. You deposit \$700 in an account that pays 8% interest compounded yearly. Find the balance after 13 years.

2. How much must you deposit in an account that pays 8% interest compounded yearly to have a balance of \$2000 after 6 years?

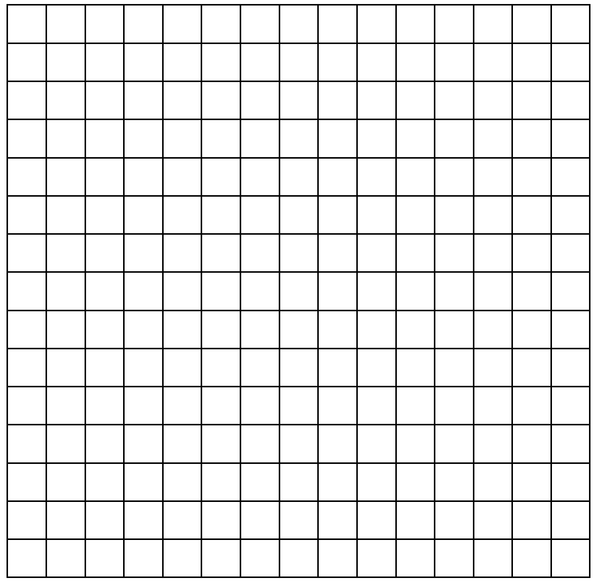
3. Company A had a \$15,000 profit. Then the profit increased by 20% per year for the next 15 years. Company B had a \$25,000 profit. Then the profit increased by 16% per year for the next 15 years. Write an exponential growth model for each situation.

Entry Ticket 6

Name____________________

A business had an $11,000 profit in 1995. Then the profit increased by 15% per year for the next 5 years.

1. Write an exponential growth model.



2. Graph the exponential growth model

Entry Ticket 7

Name____________________

Classify the model as exponential growth or exponential decay. Identify the growth or decay factor and the percent increase or decrease per time period.

1. $y=3(1.7)^t$

2. $y= 10(0.2)^t$

3. $y= 2(\frac{1}{2})^{t}$

4. You buy a used car for $10,000. It depreciates at the rate of 20% per year. Find the value of the car after 5 years?

Entry Ticket 8

Name____________________

Graph

1. $y= 3(1.4)^t$

2. $y=3(0.4)^t$

APPENDIX F

Treatment group entry tickets

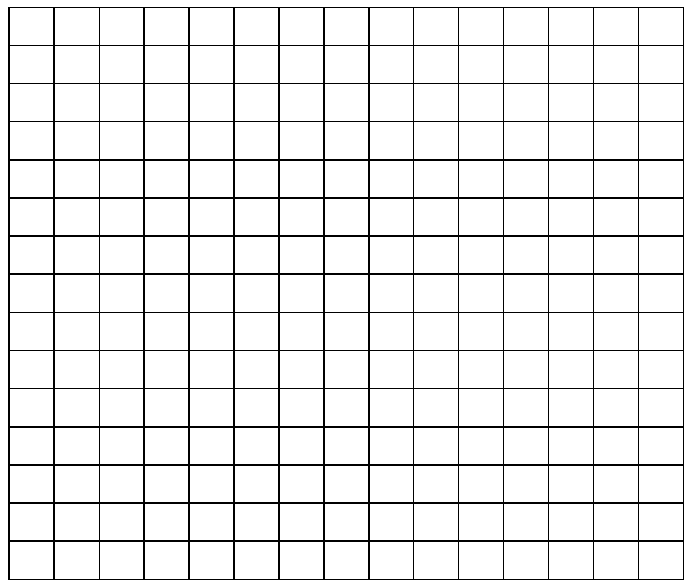
<u>Entry Ticket 1</u>

Name________________

Ray is a contestant on a Partner Quiz show. Every time he answers a question correctly, his winnings double. If he answers the first question correctly, his winnings are $1,000; if he answers the second question correctly, his winnings increase to $2,000; for the third correct answer it is $4,000, for the fourth correct answer it is $8,000, and so on.

1. Complete the table to show Ray's winnings after each correct answer.

Correct Answers	Winnings
1	
2	
3	
4	
5	
6	
7	



2. On the grid above, graph the (Correct Answers, Winnings) data from the table.

3. Write an equation for the relationship between the number of correct answers *c* and the winnings *w*.

4. How many questions must Ray answer correctly to win $128,000?

B. Simplify

1. $5^3 \times 5^7$

2. $4^3 \times 7^2 \times 4^9 \times 7^5$

Entry Ticket 2

Name_______________

The tables below represent three savings plans.

- Cela receives $20 for her birthday on January 1, puts it in her drawer, and adds $4 to it every month.
- Beginning in January, Larry hides $20 under his mattress every month.
- Noah deposits $20 in a savings account at the beginning of January and makes no more deposits. The bank adds interest to his account at a rate of 1.2% per month.

Plan 1

Month (m)	Amount (A)
0 Jan	$20
1 Feb	$40
2 Mar	$60
3 April	$80

Plan 2

Month (m)	Amount (A)
0 Jan	$20
1 Feb	$20.24
2 Mar	$20.48
3 April	$20.73

Plan 3

Month (m)	Amount (A)
0 Jan	$20
1 Feb	$24
2 Mar	$28
3 April	$32

1. Whose plan is plan 1?

2. How long does it take for the original amount of money to double in plan 1?

3. Write an equation to model the growth in plan 1.

4. How long does it take for the original amount of money to double in plan 2?

5. Write an equation to model the growth in plan 2.

6. Whose plan is plan 3?

7. How long does it take for the original amount of money to double in plan 3?

8. Write an equation to model the growth in plan 3.

Entry Ticket 3

Name_______________

On the first day of school, you notice a few patches of fungus on the leaves of the pumpkin vines in your garden. You estimate the area covered by the fungus and find that the patches cover about 1 cm^2. Suppose that the leaf area covered by this kind of fungus quadruples every day.

1. Complete the table to show what will happen to the fungus on the leaves during the first week of school.

Day	Area of Fungus(cm^2)
1	
2	
3	
4	
5	

2. On the grid above, graph the (day, fungus area) data from the table.

3. Write an equation for the fungus area *A* after *d* days.

4. How much area will the fungus cover after 8 days? Explain how you found your answer.

5. After how many days will the fungus cover at least 1,000,000 cm^2 ?

Entry Ticket 4

Name_______________

Study the pattern in the table. Tell whether the relationship between x and y is linear, exponential, or neither, and explain your answer. If the relationship is linear or exponential, write an equation for it.

1.

x	0	1	2	3	4	5
y	2	9	16	23	30	37

2.

x	0	1	2	3	4	5
y	2	4	8	16	32	64

3.

x	0	1	2	3	4	5
y	1	4	8	32	64	256

4.

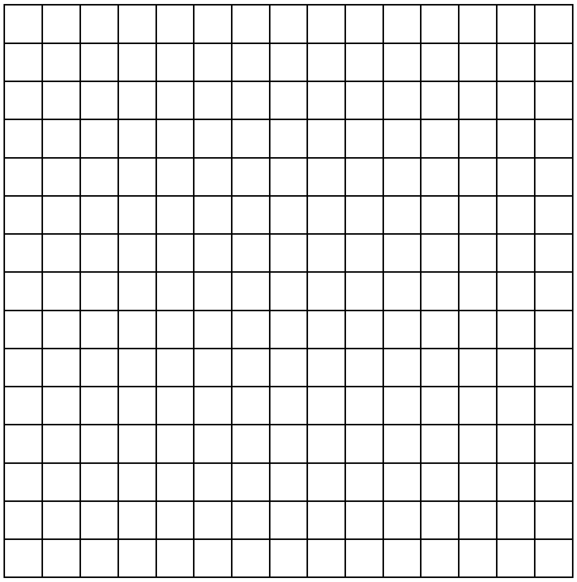
x	0	1	2	3	4	5
y	500	550	605	665.5	732.05	805.255

Entry Ticket 5

Name_______________

1. Use tables or graphs to compare these two equations for x values from 1 to 10:

$$y = 2\left(3^x\right) \qquad y = 64\left(1.5^x\right)$$



a. In which equation does the y value increase at a faster rate? How do you know?

b. For what x value are the y values equal?

2. Use the three tables below to answer the following questions.

Table 1

Year	Antelope
1997	1,000
1998	1,030
1999	1,061
2000	1,093

Table 2

Year	Antelope
1997	1,000
1998	1,030
1999	1,060
2000	1,090

Table 3

Year	Antelope
1997	1,000
1998	1,003
1999	1,006
2000	1,009

a. Which table shows a population of antelope growing at a rate of 3% per year?

b. Which table shows a population of antelope growing at a rate of 30 antelope per year?

c. Describe the growth represented in the remaining table.

d. Are any of these relationships linear? Explain.

Entry Ticket 6

Name_______________

a A sheet of paper with an area of 1 square unit is folded into thirds, and then thirds again, and so on. In the table, record the area of a region after each fold.

Number of Folds	Area of a Region
0	1
1	
2	
3	
4	
5	

b. Describe the pattern of change in the table.

c. Write an equation for the area of a region *A* after *n* folds

Simplify

1. $(2x^5)\ (3x^6)$.

2. $(3a^2m)\ (8a^5m^8)$.

3. $(4b^5)^2$.

4. $(3x^3b^2)^2\ (2a^2bm^4)^{-3}$.

5. $(2^2)^3$

6. $\left(\frac{5x^2y^2}{2x^8y^{-3}}\right)^{-2}$

Entry Ticket 7

Name_______________

A tree farm has begun to harvest a section of trees that was planted a number of years ago. The table shows the number of trees remaining for each of 8 years of harvesting.

Year	0	1	2	3	4	5	6	7	8
Trees remaining	10,000	9502	9026	8574	8145	7737	7350	6892	6543

a. Suppose the relationship between the year and the trees remaining is exponential. Approximate the decay factor for this relationship.

b. Write an equation for the relationship between time and trees remaining.

c. Evaluate your equation for each of the years shown in the table below to find the approximate number of trees remaining.

Year	10	15	20	25	30	35	40
Trees remaining							

d. The owners of the farm intend to stop harvesting when only 15% of the trees remain. During which year will this occur? Explain your reasoning.

Entry Ticket 8

Name_______________

Kai's brother collects fuzzy insects called tribetts. The tribetts population decreases by 30% each year.

a. Make a table showing the number of tribetts at the end of the first 5 years for a starting population of 10,000 tribetts.

Year	0	1	2	3	4	5
Tribetts						

b. Write an equation for the relationship between years and number of tribetts.

c. In what year will there first be fewer than 1,000 tribetts?

Decide whether each of the following statements is true or false. Explain your reasoning.

a. $25^{100} \times 25^{10} = 25^{1000}$ b. $4^9 \times 5^9 = 9^9$ c. $\left(3^6\right)^8 = 3^{48}$

d. $\frac{10^6}{10^2} = 10^3$ e. $7^0 = 1$

Entry Ticket 9

Name_______________

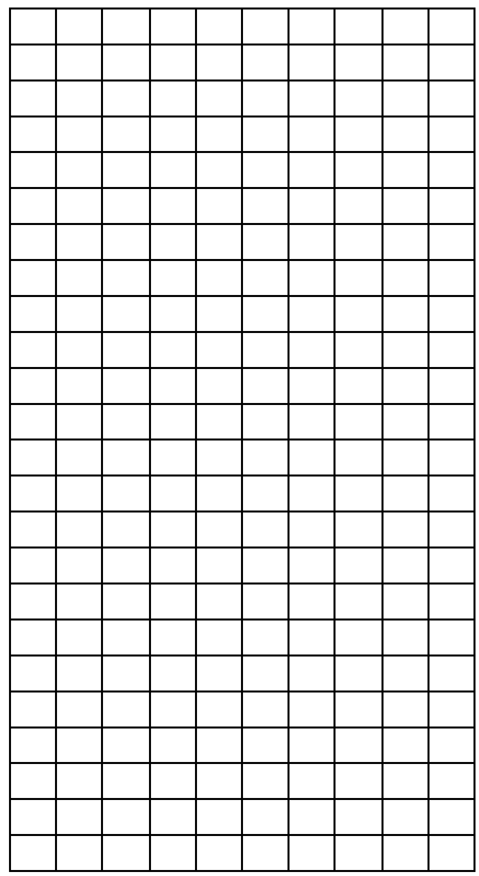
Consider these three equations: $y = 0.625^x$, $y = 0.375^x$, and $y = 1 - 0.5x$.

a. Sketch graphs of the equations on one set of axes.

b. What points, if any, do the three graphs have in common?

c. In which graph does the y-value decrease at a faster and faster rate as the x-value increases?

d. Which of the equations is not an example of exponential decay? Use the graphs or the equations to answer this question.



Entry Ticket 10

Name_______________

1. Dominic's father is a scientist who works with radioactive substances. He had a 512-gram sample of one substance, which decays every hour. Dominic's father monitored the sample every hour to determine how much radioactive material remained. The following chart shows his observations. How much would be expected after 5 hours?

Time (hours)	0	1	2	3	4	5
Mass (grams)	512	256	128	64	32	

2. Identify the initial amount *a* and the growth factor *b* in the exponential function.
   $A(x) = 680\ (4.3)^x$

3. You deposit $500 in an account that earns 5% compounded annually (once per year). What is the balance in your account after 5 years? Round your answer to the nearest cent.

4. A boat costs $15,500and decreases in value by 10% per year. How much will the boat be worth after 5 years

APPENDIX G

Students' reflection sheet

Reflection Worksheet

1. How do you feel about this type of math and problem solving? Why?

2. How would you rate your problem solving ability **before** this unit?

3. How would you rate your problem solving ability **after** this unit?

4. What skills do you think you learned or improved on by doing this type of math? Explain

5. Do you think your attitude toward problem solving or confidence in your ability to problem solve has changed? In what way?

6. What affect, if any, do you think these problem solving sessions had on your homework/test performances?

APPENDIX H

Mathematical reflections

Mathematical Reflection 1

Name_____________

1.
   a) Describe some of the rules for operating with exponents.

   b) What is scientific notation? What are its practical applications?

2. Describe the effects of a and b on the graph of y = a(bx).

3. Compare exponential and linear functions. Include in your comparison information about their patterns of change, *y*-intercepts, whether the function is decreasing or increasing, and any other information you think is important. Include examples of how they are useful.

Mathematical Reflection 2

Name______________

1. Describe an exponential growth pattern. Include key properties such as growth factors.

2. How are exponential functions similar to and different from the linear functions you worked with in earlier Units?

Mathematical Reflection 3

Name______________

1. How can you use a table, a graph, and an equation that represent an exponential function to find the *y*-intercept and growth factor for the function? Explain.

2. How can you use the *y*-intercept and growth factor to write an equation that represents an exponential function? Explain.

3. How would you change your answers to Questions 1 and 2 for a linear function?

Mathematical Reflection 4

Name______________

1. Suppose you know the initial value for a population and the yearly growth rate.
   a. How can you determine the population several years from now?

   b. How is a growth rate related to the growth factor for the population?

   c. How can you use this information to write an equation that models the situation?

2. Suppose you know the initial value for a population and the yearly growth factor.
   a. How can you determine the population several years from now?

   b. How can you determine the yearly growth rate?

3. Suppose you know the equation that represents the exponential function relating the population *p* and the number of years *n*. How can you determine the doubling time for the population?

<u>Mathematical Reflection 5</u>

Name______________

1. How can you recognize an exponential decay pattern from the following?
   a) a table of data

   b) a graph

   c) an equation

2. How are exponential growth functions and exponential decay functions similar? How are they different?

3. How are exponential decay functions and decreasing linear functions similar? How are they different?

APPENDIX I

Classroom Observation

Name ______________________________ Date___________________________

Class Observed_______________________ Time___________________________

Observer____________________________ Department______________________

**All items marked Not Observed must be explained in Comments*

Class Structure	Could Improve	Acceptable	Excellent	Not Observed
Reviews previous day's course content				
Gives overview of day's course content				
Summarizes course content covered				
Directs student preparation for next class				
Comments				

Methods	Could Improve	Acceptable	Excellent	Not Observed
Provides well-designed materials				
Employs non-lecture learning activities (i.e. small group discussion, student-led activities)				
Invites class discussion				
Employs other tools/instructional aids (i.e. technology, computer, video, overheads)				
Delivers well-planned lecture				
Comments				

Teacher-Student Interaction	Could Improve	Acceptable	Excellent	Not Observed
Solicits student input				
Involves a variety of students				
Demonstrates awareness of individual student learning needs				
Comments				

Content	Could Improve	Acceptable	Excellent	Not Observed
Appears knowledgeable				
Appears well organized				
Explains concepts clearly				
Relates concepts to students' experience				
Selects learning experiences appropriate to level of learning				
Comments				

Other Comments *-Note either effective or ineffective teaching practices observed -Attach additional pages if necessary*

Observer Signature

APPENDIX J

Quizzes

Quiz 1

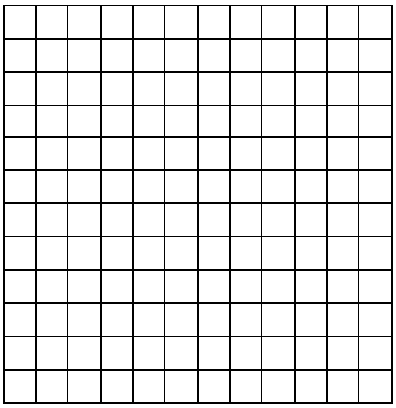
(for the experimental group)

Name____________

1. Ray is a contestant on a Partner Quiz show. Every time he answers a question correctly, his winnings double. If he answers the first question correctly, his winnings are $1,000; if he answers the second question correctly, his winnings increase to $2,000; for the third correct answer it is $4,000, for the fourth correct answer it is $8,000, and so on.

   a. Complete the table to show Ray's winnings after each correct answer.

Correct answer	Winnings



   b. On the grid above, graph the (Correct Answers, Winnings) data from the table.

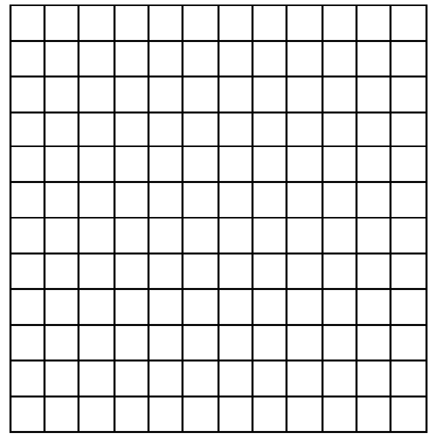
   c. Write an equation for the relationship between the number of correct answers *c* and the winnings *w*.

   d. How many questions must Ray answer correctly to win $128,000?

2. On the first day of school, you notice a few patches of fungus on the leaves of the pumpkin vines in your garden. You estimate the area covered by the fungus and find that the patches cover about 1 cm^2. Suppose that the leaf area covered by this kind of fungus quadruples every day.

   a. Complete the table to show what will happen to the fungus on the leaves during the first week of school.

Day	Area of Fungus(cm^2)
1	
2	
3	
4	
5	



   b. On the grid above, graph the (day, fungus area) data from the table.

   c. Write an equation for the fungus area *A* after *d* days.

   d. How much area will the fungus cover after 8 days? Explain how you found your answer.

   e. After how many days will the fungus cover at least 1,000,000 cm^2 ?

3.
   a. How are the patterns of change in parts (A) and (B) alike?

   b. How are the patterns of change in parts (A) and (B) different

4. Write the expression in standard form.
   a. 12^5 b. 12.5^5

Quiz 2

(for the experimental group)

Name____________

1. The tables below represent three savings plans.
   - Celia receives $20 for her birthday on January 1, puts it in her drawer, and adds $4 to it every month.
   - Janet receives $20 for her birthday on January 1, puts it in her drawer, and
   - Karen receives $20 for her birthday on January 1, puts it in her drawer, and adds $4no more deposits. The bank adds interest to his account at a rate of 1.2% per month.

Plan 1

Month (*m*)	Amount (*A*)
0 Jan	$20
1 Feb	$40
2 Mar	$60
3 April	$80

Plan 2

Month (*m*)	Amount (*A*)
0 Jan	$20
1 Feb	$20.24
2 Mar	$20.48
3 April	$20.73

Plan 3

Month (*m*)	Amount (*A*)
0 Jan	$20
1 Feb	$24
2 Mar	$28
3 April	$32

a. Whose plan is plan 1?

b. How long does it take for the original amount of money to double in plan 1?

c. Write an equation to model the growth in plan 1.

d. Whose plan is plan 2?

e. How long does it take for the original amount of money to double in plan 2?

f. Write an equation to model the growth in plan 2.

g. Whose plan is plan 3?

h. How long does it take for the original amount of money to double in plan 3?

i. Write an equation to model the growth in plan 3.

2. The table below shows an exponential pattern.

x	0	1	2	3	4	5
y	1	6	36	216	1,296	

a. Continue the table by giving the values for the next column.

b. Write an equation that represents the pattern in the table.

c. What is the growth factor? Explain how you determined the growth factor.

Quiz 1

(for the control group)

Name___________

1. Simplify $(2x^2y)^3(-3xy^2)^2$

2. Which is larger 4^3 or 3^4

3. Solve the equation for x. $(4^2)^5= 4^x$

4. Evaluate the expression $6^{-7} \bullet 6^{10}$

5. Rewrite the expression using positive exponents. $(-5y)^{-3}$

6. Evaluate $(4y)(7^0)$

7. Evaluate the expression $(-\frac{2}{5})^3$

8. Simplify $\frac{2xy^4}{5xy^2} \quad \frac{-30xy}{2x^2y}$

Quiz 2

(for the control group)

Name___________

1. Rewrite 8.67×10^{-7} in decimal form.

2. Rewrite 73,480,000 in scientific notation.

3. Evaluate the expression $(1.2 \times10^{-4})(3\times10^{6})$

4. You deposited $1000 in an account that pays 7.5% annual interest compounded yearly. What is the account balance after 5 years? Round your answer to the nearest cent.

5. A town has a population of 29,000. The population is decreasing by 2.5% each year. At this rate, what will the population be after 10 years?

APPENDIX K

Interview Questions

1. Overall, how has your experiences in this unit affected you? Consider the following questions to help you with your response:

    a) How did it affect you in choosing your future career?
    b) How did it affect you in choosing the math classes you want to take in high school?
    c) How did it affect your confidence in solving problems?
    d) How did it affect your attitude toward mathematics?

2. What activities, lessons, method of teaching helped you learn the most? Was it taking notes, solving problems in class, discussing the problem with your group, doing homework, listening to the teacher, etc.? Why?

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ABSTRACT

THE EFFECTS OF TEACHING EXPONENTIAL FUNCTIONS USING AUTHENTIC PROBLEM SOLVING ON STUDENTS' ACHIEVEMENT AND ATTITUDE

by

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Most of the current mathematics education focuses on procedures and pays very little attention to conceptual understanding in many classrooms. NCTM (2000) called for improving students' understanding to help improve their achievement. To help prepare students for the future, they need to be taught to value mathematics and realize that it is relevant to their lives. Students need to use their mathematical knowledge to solve real life problems. This can't be achieved by focusing only on the procedures and expecting students to understand how to apply it to real life. The purpose of this study was to examine the effect of using authentic problem solving on students' achievement and attitude while studying exponential functions.

In order to achieve this purpose, a mixed design was used in the study. Two groups of Algebra 1 students from two different schools were part of the study; one group was taught exponential functions using the traditional method, while the other group used authentic problem solving to learn the unit. Pre- and post-tests, quizzes, entry tickets, mathematical reflections, and pre- and post-attitude surveys were used to obtain quantitative data; students' reflections, observations, and interviews were used to obtain qualitative data.

The quantitative data revealed that, before the unit, there was no significant difference between the two groups, and after the study, there was a significant difference in favor of the experimental group. The qualitative data revealed that the students in the experimental group were more positively affected by the teaching method than the students in the control group. Student attitude was significantly higher in before and after the study in favor of the control group, but the mean improvement in the experimental group was higher than the control group. The qualitative data also revealed that students in the experimental group improved their attitude more than those in the control group.

Using authentic problem solving helped students achieve more than the traditional teaching method did. Students' attitudes were positively affected by using authentic problem solving. Further research is recommended to determine the impact of using authentic problem solving to teach other mathematics topics.

AUTOBIOGRAPHICAL STATEMENT

Yamamah Sawalha was born in Nablus, Palestine. In 1988 she finished a B.S. in Mathematics, and in 1990 she finished her M.A. degree in Mathematics from Yarmouk University in Jordan. Yamamah moved to Ann Arbor, Michigan in 1991 where she started her family. She attended Wayne State University and earned her teaching certificate in 2005, and began teaching. In 2009, she decided to return to Wayne State University and pursue a Ph.D. and a Masters degree in Curriculum and Instructions. Yamamah currently lives in Canton, Michigan with her husband and five children. She teaches high school Mathematics and is an adjunct faculty member at Madonna University.